Pg86 Q26.

$$\boxed{\int \frac{dx}{(x+2)(x^2+1)}}$$

$$\frac{A}{(x+2)} + \frac{(Bx+C)}{(x^2+1)} = \frac{1}{(x+2)(x^2+1)}$$

$$\boxed{\therefore\ A(x^2+1) + (Bx+C)(x+2) = 1}$$

Let $x=-2$:

$5A = 1$

$A = 1/5$

Let $x=0$:

$A + 2C = 1$

$2C = 1 - 1/5$

$2C = 4/5$

$\therefore\ C = 2/5$

Let $x=1$:

$2A + (B+C)(3) = 1$

$2(1/5) + 3B + \frac{6}{5} = 1$

$3B = -3/5$

$B = -1/5$

$$\int \frac{dx}{(x+2)(x^2+1)} = \int \left(\frac{1/5}{(x+2)} + \frac{(-1/5 x + 2/5)}{(x^2+1)} \right) dx$$

$$= \frac{1}{5} \int \left(\frac{1}{(x+2)} + \frac{2-x}{(x^2+1)} \right) dx$$

$$= 1/5 \int \frac{dx}{(x+2)} + \frac{1}{5} \int \frac{2\,dx}{(x^2+1)} - \frac{1}{5} \int \frac{x\,dx}{(x^2+1)}$$

Let $u = x^2+1$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$$= \frac{1}{5} \int \frac{dx}{(x+2)} + \frac{2}{5} \int \frac{dx}{(x^2+1)} - \frac{1}{5} \int \frac{x}{u} \cdot \frac{du}{2x}$$

$$= 1/5 \left[\int \frac{dx}{(x+2)} + 2 \int \frac{dx}{(x^2+1)} - 1/2 \int \frac{du}{u} \right]$$

$$= 1/5 \left[\ln|x+2| + 2\tan^{-1}(x) - 1/2 \ln|u| \right] + C$$

$$= 1/5 \left[\ln|x+2| + 2\tan^{-1}(x) - \ln|(x^2+1)|^{1/2} \right] + C$$

[Book made an error]

$$= 1/5 \left[\ln \left| \frac{(x+2)}{\sqrt{x^2+1}} \right| + 2\tan^{-1}(x) \right] + C$$

or

$$\ln \left| \frac{x+2}{\sqrt{x^2+1}} \right|^{1/5} + \frac{2}{5} \tan^{-1}(x) + C$$

Pg 86 Unit 2 Ch 3 Review Qs Requested

2d)

$$\int \frac{5x+2}{(x+1)(x^2+x+5)}\,dx$$

$$\frac{A}{(x+1)} + \frac{(Bx+C)}{(x^2+x+5)} = \frac{(5x+2)}{(x+1)(x^2+x+5)}$$

$$\boxed{A(x^2+x+5) + (x+1)(Bx+C) = 5x+2}$$

Let $x=-1$:

$A(1-1+5) = -5+2$

$5A = -3$

$\therefore \underline{A = \frac{-3}{5}}$

Let $x=0$:

$5A + C = 2$

$5\left(\frac{-3}{5}\right) + C = 2$

$-3 + C = 2$

$\therefore \underline{C = 5}$

Let $x=1$:

$7A + 2B + 2C = 7$

$-\frac{21}{5} + 2B + 10 = 7$

$-21 + 10B + 50 = 35$

$10B = 6$

$\therefore \underline{B = 3/5}$

$$= \int \left(\frac{-\frac{3}{5}}{(x+1)} + \frac{\left(\frac{3}{5}x+5\right)}{(x^2+x+5)} \right) dx$$

$$= \frac{3}{5}\int \left(\frac{-1}{(x+1)} + \frac{x+\frac{25}{3}}{(x^2+x+5)} \right) dx$$

$$= \frac{3}{5}\int \left(\frac{-1}{(x+1)}\right) dx + \frac{3}{5}\int \frac{x+25/3}{(x^2+x+5)}\,dx$$

$$= -\frac{3}{5}\int \frac{dx}{(x+1)} + \frac{3}{5}\cdot\frac{2}{2}\int \frac{x+25/3}{(x^2+x+5)}\,dx$$

Let $u = x^2+x+5$

$\frac{du}{dx} = 2x+1$

$\frac{du}{(2x+1)} = dx$

So need to manipulate to create $(2x+1)$ to reduce down

$$= \frac{-3}{5}\int \frac{dx}{(x+1)} + \frac{3}{10}\int \frac{2x+50/3}{(x^2+x+5)}\,dx$$

$$= -\frac{3}{5}\int \frac{dx}{(x+1)} + \frac{3}{10}\int \left(\frac{2x+1}{(x^2+x+5)} + \frac{47/3}{(x^2+x+5)} \right) dx$$

Finally, complete square

x^2+x+5

$= \left(x+\frac{1}{2}\right)^2 + 5 - \frac{1}{4}$

$= \left(x+\frac{1}{2}\right)^2 + \frac{19}{4}$

$$= -\frac{3}{5}\int \frac{dx}{(x+1)} + \frac{3}{10}\int \frac{\cancel{2x+1}}{(u)}\cdot\frac{du}{\cancel{(2x+1)}} + \frac{3}{10}\int \frac{47/3}{(x^2+x+5)}\,dx$$

Can then use fact

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$= -\frac{3}{5}\int \frac{dx}{(x+1)} + \frac{3}{10}\int \frac{du}{u} + \frac{47}{10}\int \frac{dx}{(x+\frac{1}{2})^2+\frac{19}{4}}$$

$$= -\frac{3}{5}\int \frac{dx}{(x+1)} + \frac{3}{10}\int \frac{du}{u} + \frac{47}{10}\int \frac{dx}{(x+\frac{1}{2})^2+\left(\frac{\sqrt{19}}{2}\right)^2}$$

$$= \frac{-3}{5}\ln|x+1| + \frac{3}{10}\ln|u| + \frac{47}{10}\left(\frac{1}{\left(\frac{\sqrt{19}}{2}\right)} \tan^{-1}\left(\frac{x+1/2}{\frac{\sqrt{19}}{2}}\right) \right) + C$$

$$= \frac{3}{5}\left(\frac{1}{2}\ln|x^2+x+5| - \ln|x+1| \right) + \frac{47}{5\sqrt{19}}\tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right) + C$$

$$= \frac{3}{5}\left(\ln\left|\frac{\sqrt{x^2+x+5}}{x+1}\right| \right) + \frac{47\sqrt{19}}{95}\tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right) + C$$

(Textbook Incorrect)

Pg 86

2e) $\int \frac{x^3+x}{(x+1)(x-2)}\,dx$

$= \int \frac{x^3+x}{x^2-x-2}\,dx \rightarrow$

$$\begin{array}{r|l} & x+1 \\ \hline x^2-x-2 & x^3 \quad\quad +x \\ & x^3-x^2-2x \\ \hline & \quad x^2+3x \\ & \quad x^2-x-2 \\ \hline & \quad\quad 4x+2 \end{array}$$

$= \int \left(x+1+\frac{4x+2}{(x^2-x-2)}\right)dx$

$= \int \left(x+1+\frac{4x+2}{(x+1)(x-2)}\right)dx$

$\frac{4x+2}{(x+1)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x-2)}$

$4x+2 = A(x-2)+B(x+1)$

$x=-1$:

$-4+2=-3A$

$-2=-3A$

$\Rightarrow A = 2/3$

$x=2$:

$8+2=3B$

$10=3B$

$\Rightarrow B = \frac{10}{3}$

$= \int \left(x+1+\frac{2/3}{(x+1)}+\frac{10/3}{(x-2)}\right)dx$

$= \frac{x^2}{2} + x + \frac{2}{3}\ln|x+1| + \frac{10}{3}\ln|x-2| + C$

$= \frac{x^2}{2} + x + \frac{2}{3}\left(\ln|x+1| + 5\ln|x-2|\right) + C$

OR $= \frac{x^2}{2} + x + \frac{2}{3}\ln|(x+1)(x-2)^5| + C$

(Textbook Incorrect)

Q3. b) $\int_0^{\pi/4} 3x^2 \sin 2x \, dx$

Let $u = 3x^2$ $\quad v = \frac{\cos 2x}{-2}$

$u' = 6x$ $\quad v' = \sin 2x$

$$\boxed{\int uv' = uv - \int u'v}$$

$$\int_0^{\pi/4} 3x^2 \sin 2x \, dx = \left[3x^2 \cdot \frac{\cos 2x}{-2}\right]_0^{\pi/4} - \int_0^{\pi/4} \left(6x \cdot \frac{\cos 2x}{-2}\right) dx$$

$u = 3x$ $\quad v = \frac{\sin 2x}{2}$

$u' = 3$ $\quad v' = \cos 2x$

$$= \left[-\frac{3x^2 \cos 2x}{2}\right]_0^{\pi/4} + \int_0^{\pi/4} 3x \cos 2x \, dx$$

$$= \left[-\frac{3x^2 \cos 2x}{2}\right]_0^{\pi/4} + \left[\left[\frac{3x \sin 2x}{2}\right]_0^{\pi/4} - \int_0^{\pi/4} \frac{3}{2} \sin 2x \, dx\right]$$

$$= \left[-\frac{3x^2 \cos 2x}{2} + \frac{3x \sin 2x}{2} + \frac{3}{2}\left(\frac{\cos 2x}{2}\right)\right]_0^{\pi/4}$$

$$= \left(\frac{-3\left(\frac{\pi}{4}\right)^2 \cos\left(\frac{\pi}{2}\right)}{2} + \frac{3\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{2}\right)}{2} + \frac{3}{4}\cos\left(\frac{\pi}{2}\right)\right) - \left(0 + 0 + \frac{3}{4}\cos 0\right)$$

$$= \left(0 + \frac{\frac{3\pi}{4} \times 1}{2} + 0\right) - \left(\frac{3}{4} \times 1\right)$$

$$= \frac{3\pi}{8} - \frac{3}{4}$$

$$= \frac{3\pi - 6}{8}$$

$$= \frac{3(\pi - 2)}{8}$$

(Again textbook Answer Incorrect)

Q5. $\frac{dy}{dx} = 4y^2e^{2x}$

$$\frac{dy}{y^2} = 4e^{2x}\,dx$$

$$\int y^{-2}\,dy = \int 4e^{2x}\,dx$$

$$\frac{y^{-1}}{-1} = \frac{4e^{2x}}{2} + c$$

$$-\frac{1}{y} = 2e^{2x} + c$$

$\therefore\ y = \dfrac{-1}{(2e^{2x}+c)}$ is General Solution.

$\left.\begin{array}{l} x=0 \\ y=-1/2 \end{array}\right]$ $\quad -\frac{1}{2} = \frac{-1}{2e^{0}+c}$

$$-\frac{1}{2} = \frac{-1}{2+c}$$

$\Rightarrow\ c = 0$

$\therefore\ y = \dfrac{-1}{2e^{2x}}$

or $y = -\frac{1}{2}e^{-2x}$

[Error in Book Again].

Q7. $m = \dfrac{dy}{dx} = \dfrac{-3x}{\sqrt{1-x^2}}$

Let $u = 1-x^2$

$\dfrac{du}{dx} = -2x$

$\therefore \dfrac{du}{-2x} = dx$

$$\frac{dy}{dx} = \frac{-3x}{\sqrt{1-x^2}}$$

$$y = \int \frac{-3x}{\sqrt{1-x^2}} \cdot dx$$

$$y = \int \frac{-3\cancel{x}}{\sqrt{u}} \cdot \frac{du}{-2\cancel{x}}$$

$$y = \frac{3}{2}\int \frac{du}{\sqrt{u}}$$

$$= \frac{3}{2}\int u^{-1/2}\, du$$

$$= \frac{3}{2}\left(\frac{u^{1/2}}{1/2}\right) + c$$

$$y = 3\sqrt{u} + c$$

$$\therefore \quad y = 3\sqrt{1-x^2} + c$$

NB Ellipses not in AH Course, so you would not identify this, but still expect to find a PARTICULAR SOLUTION

Taking further:

$$y - c = 3\sqrt{1-x^2}$$

$$\frac{y-c}{3} = \sqrt{1-x^2}$$

$$\frac{(y-c)^2}{9} = 1 - x^2$$

$$\therefore \quad x^2 + \frac{(y-c)^2}{9} = 1$$

Ellipses have equations:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Family of Ellipses

$\Rightarrow$ Centre $(0, c)$