Q56 (g156)

$$y = \frac{x^2 - 3x + 3}{x - 1}$$

Axes: $x = 0$: $y = \frac{3}{-1} = -3 \Rightarrow (0, -3)$

$y = 0$: $\frac{x^2 - 3x + 3}{x - 1} = 0$

$$x^2 - 3x + 3 = 0$$

$$b^2 - 4ac = (-3)^2 - 4 \times 1 \times 3 = 9 - 12 < 0$$

As $b^2 - 4ac < 0 \Rightarrow$ Doesn't cut x-axis

Symmetry: $\frac{(-x)^2 - 3(-x) + 3}{(-x) - 1} = \frac{x^2 + 3x + 3}{-x - 1} = \frac{x^2 + 3x + 3}{-(x + 1)}$

As $f(-x) \neq \pm f(x) \Rightarrow$ Not Odd/Even
$\Rightarrow$ No Symmetry

Max/Min Pts

$$y = \frac{x^2 - 3x + 3}{x - 1}$$

$$y = x - 2 + \left(\frac{1}{x - 1}\right) \quad \longleftrightarrow$$

$$y = x - 2 + (x - 1)^{-1}$$

$$x - 2 \text{ remainder } 1$$

$$\frac{dy}{dx} = 1 - (x - 1)^{-2}$$

$$= 1 - \frac{1}{(x - 1)^2}$$

Stat Pts $\Rightarrow$ $dy/dx = 0$

$$1 - \frac{1}{(x - 1)^2} = 0$$

$$1 = \frac{1}{(x - 1)^2}$$

$$(x - 1)^2 = 1$$

2 Stat Pts at.

$$x - 1 = \pm 1$$

$$x = 1 \pm 1 \Rightarrow x = 0 \text{ or } x = 2$$

256 $y = \frac{x^2-3x+3}{x-1} = x-2+\frac{1}{(x-1)}$ (2)

2 Stat Pts

$x=0$: $y = \frac{3}{-1} = -3 \Rightarrow (0,-3)$

$x=2$: $y = \frac{(2)^2-3(2)+3}{(2)-1} = \frac{1}{1} = 1 \Rightarrow (2,1)$

Nature: $\frac{dy}{dx} = 1-(x-1)^{-2}$

$\frac{d^2y}{dx^2} = 2(x-1)^{-3} = \frac{2}{(x-1)^3}$

At $x=0$ $\frac{d^2y}{dx^2} = \frac{2}{(0-1)^3} = -2 < 0 \Rightarrow$ ∩ Max TPt $(0,-3)$

At $x=2$: $\frac{d^2y}{dx^2} = \frac{2}{(2-1)^3} = 2 > 0 \Rightarrow$ ∪ Min TPt $(2,1)$

POI $\left|\frac{d^2y}{dx^2}=0\right|$ $\frac{2}{(x-1)^3} = 0$

$2 \neq 0 \Rightarrow$ No POI exist

Extremes of Asymptotes

Vertical Asymptote at $x=1$ (Undefined)

$x \to 1^+$	$y \to +\infty$
$x \to 1^-$	$y \to -\infty$

Non-horizontal Asymptote $y = x-2$ (as $x \to \pm\infty$)

for $y = x-2+\frac{1}{(x-1)}$

$x \to +\infty$	$y \to (x-2)^+$
$x \to -\infty$	$y \to (x-2)^-$

Q56 Summary Max (0, -3) ; Mii (2, 1)

$x = 1$ & $y = x - 2$ are 2 asymptotes