6) $y = \frac{x^2 - 2x}{x^2 - 1}$ (1)

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ces: $x = 0$: $y = \frac{0}{-1} = 0 \Rightarrow (0,0)$

$y = 0$: $\frac{x^2 - 2x}{x^2 - 1} = 0$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0 \quad x = 2 \Rightarrow (0,0) \& (2,0)$

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mmetry $\frac{(-x)^2 - 2(-x)}{(-x)^2 - 1} = \frac{x^2 + 2x}{x^2 - 1}$

$f(-x) \neq \pm f(x)$
$\Rightarrow$ No Symmetry

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x/Min
pts $\frac{dy}{dx} = \frac{(2x-2)(x^2-1) - 2x(x^2-2x)}{(x^2-1)^2}$

$u = x^2 - 2x \quad v = x^2 - 1$
$u' = 2x - 2 \quad v' = 2x$

$\frac{dy}{dx} = \frac{2x^3 - 2x - 2x^2 + 2 - 2x^3 + 4x^2}{(x^2-1)^2}$

r Pts $= 0$ $\Rightarrow \frac{2x^2 - 2x + 2}{(x^2-1)^2} = 0$

$2x^2 - 2x + 2 = 0$

$x^2 - x + 1 = 0$

$b^2 - 4ac = (-1)^2 - 4 \times 1 \times 1$
$= 1 - 4$
$= -3$

As $b^2 - 4ac < 0$
$\Rightarrow$ No Stat Pts exist
As $\frac{dy}{dx} = 0$ has no solution

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56) $\frac{dy}{dx} = \frac{2x^2 - 2x + 2}{(x^2-1)^2}$

$u = 2x^2 - 2x + 2$

$u' = 4x - 2$

$v = (x^2-1)^2$

$v' = 2(x^2-1)^1 \cdot 2x$

$= 4x(x^2-1)$

$$\frac{d^2y}{dx^2} = \frac{(4x-2)(x^2-1)^2 - (2x^2-2x+2)4x(x^2-1)}{(x^2-1)^4}$$

$$= \frac{(4x-2)(x^2-1) - 4x(2x^2-2x+2)}{(x^2-1)^3}$$

$$= \frac{4x^3 - 4x - 2x^2 + 2 - 8x^3 + 8x^2 - 8x}{(x^2-1)^3}$$

$$= \frac{-4x^3 + 6x^2 - 12x + 2}{(x^2-1)^3}$$

$\frac{d^2y}{dx^2} = 0$ for POI to exist

$$\frac{-4x^3 + 6x^2 - 12x + 2}{(x^2-1)^3} = 0$$

$$-4x^3 + 6x^2 - 12x + 2 = 0$$

$$-2(2x^3 - 3x^2 + 6x - 1) = 0$$

	2	−3	6	−1
1	↓	2	−1	5
	2	−1	5	X

	2	−3	6	−1
−1	↓	−2	5	−11
	2	−5	11	X

As cubic doesn't factorise ⇒ No POI

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$$y = \frac{x^2-2x}{x^2-1} = \frac{x^2-2x}{(x-1)(x+1)}$$

ctremes of
Asymptotes

2 Vertical Asymptotes exist at $x = \pm 1$

$x \to 1^+$	$y \to \infty^-$	$x \to -1^+$	$y \to \infty^-$
$x \to 1^-$	$y \to \infty^+$	$x \to -1^-$	$y \to \infty^+$

Horizontal Asymptote $(x \to \pm\infty)$

$$y = \frac{x^2-2x}{x^2-1} = \frac{1-2/x}{1-1/x^2}$$

$x \to +\infty$	$y \to 1^-$
$x \to -\infty$	$y \to 1^+$

Summary (0,0), (2,0) No Max/Min/POI