$$y = \frac{4x-5}{x^2-1} = \frac{4x-5}{(x-1)(x+1)}$$

---

es $x=0$: $y = \frac{-5}{-1} = 5 \Rightarrow (0,5)$

$y=0$: $0 = \frac{4x-5}{x^2-1}$

$0 = 4x-5$

$4x = 5$

$x = 5/4 \Rightarrow (\frac{5}{4}, 0)$

---

mmetry $\frac{4(-x)-5}{(-x)^2-1} = \frac{-4x-5}{x^2-1} = -\frac{(4x+5)}{x^2-1}$

As $f(-x) \neq \pm f(x) \Rightarrow$ Not Odd / Even
$\Rightarrow$ No Symmetry

Nu
Pts

$u = 4x-5 \quad v = x^2-1$
$u' = 4 \quad v' = 2x$

$\frac{u'v-uv'}{v^2}$

$\frac{dy}{dx} = \frac{4(x^2-1) - 2x(4x-5)}{(x^2-1)^2} = \frac{4x^2-4-8x^2+10x}{(x^2-1)^2}$

Pts
$=0$

$\Rightarrow \frac{-4x^2+10x-4}{(x^2-1)^2} = 0$

$-4x^2+10x-4 = 0$

$-2(2x^2-5x+2) = 0$

$(2x-1)(x-2) = 0$

$x = 1/2$ & $x = 2$

Stat Pts
ist at

$(1/2, 4)$ & $(2, 1)$

$y = \frac{4x-5}{x^2-1}$

$x = \frac{1}{2}$: $y = \frac{4(1/2)-5}{(1/2)^2-1}$

$= \frac{-3}{-3/4}$

$= 4$

$x = 2$ $y = \frac{4(2)-5}{(2)^2-1} = \frac{3}{3} = 1$

$$\frac{dy}{dx} = \frac{-4x^2+10x-4}{(x^2-1)^2}$$

(2)

2 Stat Pts at $(\frac{1}{2},4)$ & $(2,1)$

Nature table or $\frac{d^2y}{dx^2}$

$u = -4x^2+10x-4 \qquad v = (x^2-1)^2$

$u' = -8x+10 \qquad v' = 2(x^2-1)^1 \cdot 2x = 4x(x^2-1)$

$$\frac{d^2y}{dx^2} = \frac{(-8x+10)(x^2-1)^2 - 4x(x^2-1)(-4x^2+10x-4)}{((x^2-1)^2)^2}$$

$$= \frac{(-8x+10)(x^2-1) - 4x(-4x^2+10x-4)}{(x^2-1)^3}$$

$$= \frac{-8x^3+8x+10x^2-10+16x^3-40x^2+16x}{(x^2-1)^3}$$

$$= \frac{8x^3-30x^2+24x-10}{(x^2-1)^3}$$

$x = \frac{1}{2}$: $\frac{d^2y}{dx^2} = \frac{8(\frac{1}{8})-30(\frac{1}{4})+24(\frac{1}{2})-10}{(\frac{1}{4}-1)^3} = \frac{32}{3} > 0 \cup$ MIN

$x = 2$: $\frac{d^2y}{dx^2} = \frac{8(8)-30(4)+24(2)-10}{(4-1)^3} = -\frac{2}{3} < 0 \cap$ Max

ii Min TPt $(\frac{1}{2},4)$ & Max TPt $(2,1)$

52

POI $\quad \frac{d^2y}{dx^2} = 0$ $\quad \frac{8x^3 - 30x^2 + 24x - 10}{(x^2-1)^3} = 0$

Doesn't factorise $\quad 8x^3 - 30x^2 + 24x - 10 = 0$

So no POI exist. $\quad 4x^3 - 15x^2 + 12x - 5 = 0$

1 | 4 −15 12 −5
↓ 4 −11 1
4 −11 1 X

5 | 4 −15 12 −5
↓ 20 25
4 5 37 X

1 | 4 −15 12 −5
↓ −4 19
4 −19 31 X

−5 | 4 −15 12 −5
↓ −20 175
4 −35 X

Extremes of Asymptotes

Vertical (undefined) $\quad x^2 - 1 = 0$

$x^2 = 1$

Vertical when $x = \pm 1$

$x \to 1^+ \quad y \to \infty^-$

$x \to 1^- \quad y \to \infty^+$

$x \to -1^+ \quad y \to \infty^+$

$x \to -1^- \quad y \to \infty^-$

Horizontal Asymptote (when $x \to \pm\infty$)

$y = \frac{4x-5}{x^2-1} = \frac{4 - 5/x}{x - 1/x}$ $\left(\to \frac{4}{x} \to \frac{4}{\infty}\right)$

$x \to +\infty \quad y \to 0^+$

$x \to -\infty \quad y \to 0^-$

Summary Asymptotes at $x = \pm 1$ & $y = 0$

Min TPt $(\frac{1}{2}, 4)$ Cuts at $(0, 5^-)$ & $(\frac{5}{4}, 0)$

Max TPt $(2, 1)$