Advanced Higher : Implicit & Parametric Differentiation

Mark

1. A curve has the equation $xy + y^2 = 2$

(a) Use implicit differentiation to obtain $\frac{dy}{dx}$ in terms of ***x* & *y*** *3*

(b) Hence find an equation to the tangent of the curve at point (1, 1). *2*

2. A curve is defined by the parametric equations $x = t^2 + t - 1,\ y = 2t^2 - t + 2$ for all ***t***. Show that ***A(-1, 5)*** lies on the curve and obtain an equation of the tangent to the curve at point ***A***. *6*

3. The equation $y^3 + 3xy = 3x^2 - 5$ defines a curve passing through the point A (2, 1). Obtain an equation for the tangent to the curve at A *4*

4. A curve is defined by the equations $x = 5\cos\theta \ and \ y = 5\sin\theta \quad (0 \le \theta \le 2\pi)$

Use parametric differentiation to find $\frac{dy}{dx}$ in terms θ. *2*

Find an equation to the tangent of the curve where $\theta = \frac{\pi}{4}$ *3*

5. Given the equation $2y^2 - 2xy - 4y + x^2 = 0$ of a curve, obtain the ***x***-coordinate of each point at which the curve has a horizontal tangent. *4*

6. Given $xy - x = 4$, use implicit differentiation to obtain $\frac{dy}{dx}$ in terms of *x* & *y* *2*

Hence obtain $\frac{d^2y}{dx^2}$ in terms of *x* & *y* *3*

7. A curve is defined by the parametric equations $x = \cos 2t,\ y = \sin 2t,\ 0 < t < \frac{\pi}{2}$

(a) Use parametric differentiation to find $\frac{dy}{dx}$,

Hence find the equation of the tangent when $t = \frac{\pi}{8}$ *5*

(b) Obtain an expression for $\frac{d^2y}{dx^2}$ and hence show that

$\sin 2t \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = k,$ *where k* is an integer. State the value of *k*. *5*

8. Given $x = 2\sec\theta$, $y = 3\sin\theta$, use parametric differentiation to find $\frac{dy}{dx}$ in terms of θ. *3*

9. A curve is defined by the equation $xy^2 + 3x^2y = 4$ for $x > 0$ and $y > 0$.

Use implicit differentiation to find $\frac{dy}{dx}$. *3*

Hence find the equation of the tangent to the curve where $x = 1$. *3*

10. Calculate the gradient of the curve defined by $\frac{x^2}{y} + x = y - 5$ at the point (3, -1) *4*

11. Given $y = t^3 - \frac{5}{2}t^2$ & $x = \sqrt{t}$ for $t > 0$, use parametric differentiation to express $\frac{dy}{dx}$ in terms of t in simplified form. *4*

Show that $\frac{d^2y}{dx^2} = at^2 + bt$, determining the values of the constants *a* and *b*. *3*

Obtain an equation for the tangent to the curve which passes through the point of inflexion. *3*