Advanced Higher : Basic & Logarithmic Differentiation

Mark

1. Given $f(x) = x(1+x)^{10}$, obtain $f'(x)$ and simplify your answer. *3*

2. (a) $f(x) = (2+x)\tan^{-1}\left(\sqrt{x-1}\right)$, $x > 1$ obtain $f'(x)$. *4*

(b) $g(x) = e^{\cot 2x}$, $0 < x < \frac{\pi}{2}$, obtain $g'(x)$ and simplify. *2*

3. (a) Given $f(x) = \sqrt{x}\, e^{-x}$, $x \geq 0$, obtain and simplify $f'(x)$ *4*

(b) Given $y = (x+1)^2\,(x+2)^{-4}$ and $x > 0$,

use logarithmic differentiation to show that $\frac{dy}{dx}$ can be expressed in the

form $\left(\frac{a}{x+1} + \frac{b}{x+2}\right)y$, stating the values of the constants ***a*** and ***b.*** *3*

4. Given $y = 3^x$ use logarithmic differentiation to obtain $\frac{dy}{dx}$ in terms of x *3*

5. (a) Given $f(x) = cos^2 x\, e^{\tan x}$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, obtain $f'(x)$ *3*

and evaluate $f'\left(\frac{\pi}{4}\right)$ *1*

(b) Differentiate $g(x) = \frac{\tan^{-1}(2x)}{1+4x^2}$ *3*

6. (a) Given $f(x) = x^3\tan 2x$, $0 < x < \frac{\pi}{4}$ obtain $f'(x)$. *3*

(b) For $y = \frac{1+x^2}{1+x}$, $where\ x \neq -1$, determine $\frac{dy}{dx}$ & simplify *3*

7. Differentiate $\frac{1+\ln x}{3x}$, where $x > 0$ *3*

8. $f(x) = 2\tan^{-1}\sqrt{1+x}$, where $x > -1$ Find $f'(x)$ & simplify **3**

9. Obtain the derivative of each of the following functions:

(a) $f(x) = e^{(\sin 2x)}$; **3**

(b) $y = 4^{(x^2+1)}$ **3**

10. Differentiate $f(x) = \cos^{-1}(3x)$, where $-\frac{1}{3} < x < \frac{1}{3}$ **2**

11. The curve $y = x^{2x^2+1}$ is defined for $x > 0$. Obtain the values of y and $\frac{dy}{dx}$ at the point where $x = 1$. **5**

12. Write down the derivative of $\tan x$. **1**

Show that $1 + \tan^2 x = \sec^2 x$. Hence obtain $\int \tan^2 x \; dx$ **1, 2**

13. A body moves along a straight line with velocity $v = t^3 - 12t^2 + 32t$ at time t.

(a) Obtain the value of its acceleration when $t = 0$. **1**

(b) At time $t = 0$, the body is at the origin O. Obtain a formula for the displacement of the body at time t. **2**

Show that the body returns to O, and obtain the time, T, when this happens. **2**

14. Given $f(x) = (x+1)(x-2)^3$, obtain the values of x for which $f'(x) = 0$. **3**

15. (a) Given $f(x) = e^x \sin x^2$, obtain $f'(x)$ **3**

(b) Given $g(x) = \frac{x^3}{(1+\tan x)}$, obtain $g'(x)$ **3**