Q1. $\left(3x^2 - \frac{t}{x}\right)^6$ Independent term is 2160

$$\sum_{r=0}^{6} \binom{6}{r} (3x^2)^{6-r} \left(-\frac{t}{x}\right)^r$$

$$= \underbrace{\sum_{r=0}^{6} \binom{6}{r} (3)^{6-r} (-t)^r}_{\text{Coeff}} \; \underbrace{(x^2)^{6-r} (x^{-1})^r}$$

$x^{12-2r} \times x^{-r} = x^0$

$x^{12-3r} = x^0$

$12 - 3r = 0$

$-3r = -12$

$\therefore \; r = 4$

$r=4$: $\binom{6}{4}(3)^{6-4}(-t)^4$

$$C = \frac{6!}{4!\,2!}(3)^2 \times t^4$$

$$\frac{6 \times 5 \times \cancel{4!}}{\cancel{4!} \times 2} \times 9t^4 = 2160$$

$135t^4 = 2160$

$t^4 = 16$

$\therefore \; t = 2$

4

Q2. $y = \dfrac{\sin^{-1}(\sqrt{x})}{\sqrt{1-x}} = \dfrac{\sin^{-1}(x^{1/2})}{(1-x)^{1/2}}$

$u = \sin^{-1}(x^{1/2})$	$v = (1-x)^{1/2}$
$u' = \frac{1}{2}x^{-1/2} \cdot \dfrac{1}{\sqrt{1-(x^{1/2})^2}}$	$v' = -1 \cdot \frac{1}{2}(1-x)^{-1/2}$
$= \dfrac{1}{2\sqrt{x}\sqrt{1-x}}$	$= \dfrac{-1}{2\sqrt{1-x}}$

$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{\dfrac{1}{2\sqrt{x}\sqrt{1-x}} \times \sqrt{1-x} + \dfrac{\sin^{-1}(\sqrt{x})}{2\sqrt{1-x}}}{(\sqrt{1-x})^2}$$

$$= \frac{\dfrac{\sqrt{1-x}}{2\sqrt{x}\sqrt{1-x}} + \left(\dfrac{\sqrt{x}}{\sqrt{x}}\right)\left(\dfrac{\sin^{-1}(\sqrt{x})}{2\sqrt{1-x}}\right)}{(1-x)}$$

$$= \frac{\dfrac{(\sqrt{1-x}) + (\sqrt{x}\sin^{-1}(\sqrt{x}))}{2\sqrt{x(1-x)}}}{(1-x)}$$

$$= \frac{\sqrt{1-x} + \sqrt{x}\sin^{-1}(\sqrt{x})}{2\sqrt{x(1-x)}\,.\,(1-x)}$$

4

$$= \frac{\sqrt{1-x} + \sqrt{x}\sin^{-1}(\sqrt{x})}{2\sqrt{x(1-x)^3}}$$

As required.

Q3. $f(z) = 3z^3 - 8z^2 + 34z - 20$

$z = 1-3i$ is a root $\Rightarrow \bar{z} = 1+3i$ as a conjugate pair exists.

If $z = 1-3i$ & $z = 1+3i$

Factors are: $(z-1+3i)=0$ & $(z-1-3i)=0$

Multiplying:

	z	-1	$+3i$
z	z^2	$-z$	$\cancel{+3iz}$
-1	$-z$	$+1$	$\cancel{-3i}$
$-3i$	$\cancel{-3iz}$	$\cancel{+3i}$	$-9i^2$

$(z-1+3i)(z-1-3i)$

$= z^2 - z - z + 1 - 9i^2$

$= z^2 - 2z + 1 + 9$

$= z^2 - 2z + 10$

To find 3rd root use quadratic and divide

$$
\begin{array}{r|l}
 & 3z - 2 \\
\hline
z^2-2z+10 & 3z^3 - 8z^2 + 34z - 20 \\
 & 3z^3 - 6z^2 + 30z \\
\hline
 & -2z^2 + 4z - 20 \\
 & -2z^2 + 4z - 20 \\
\hline
 & - \quad - \quad -
\end{array}
$$

$\Rightarrow$ $3z-2$ is remaining factor.

ie. $z = 2/3$ is final root

3

$(z-1+3i)(z-1-3i)(3z-2) = 0$

$z = 1-3i$; $z = 1+3i$ & $z = 2/3$ are solutions/roots

$$x + 2y + z = 4$$
$$3x + 5y + 6z = 10$$
$$2x + 5y - z = 10$$
$$\rightarrow \begin{array}{c}\left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 3 & 5 & 6 & 10 \\ 2 & 5 & -1 & 10 \end{array}\right)\end{array} \begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array}$$

$$\begin{array}{c} 6r_1 \\ 2r_2 \\ 3r_3 \end{array} \left(\begin{array}{ccc|c} 6 & 12 & 6 & 24 \\ 6 & 10 & 12 & 20 \\ 6 & 15 & -3 & 30 \end{array}\right) \begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} \rightarrow \begin{array}{c} \\ r_2 - r_1 \\ r_3 - r_1 \end{array} \left(\begin{array}{ccc|c} 6 & 12 & 6 & 24 \\ 0 & -2 & 6 & -4 \\ 0 & 3 & -9 & 6 \end{array}\right) \begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array}$$

$$\begin{array}{c} \frac{1}{6}r_1 \\ \frac{1}{2}r_2 \\ \frac{1}{3}r_3 \end{array} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -1 & 3 & -2 \\ 0 & 1 & -3 & 2 \end{array}\right) \begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} \rightarrow \begin{array}{c} \\ \\ r_2 + r_3 \end{array} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -1 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right) \begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array}$$

As $0x + 0y + 0z = 0 \Rightarrow$ Infinite solutions exist

Let $z = k$ then $-y + 3z = -2$ $\qquad x + 2y + z = 4$

$2 + 3z = y$ $\qquad x + 2(3k+2) + k = 4$

$\therefore \; y = 3k + 2$; $\qquad x + 6k + 4 + k = 4$

$x = -7k$

Thus infinite solutions exist and for some k,

$x = -7k$; $y = 3k + 2$ & $z = k$

6

For $y = -1$ $\quad -1 = 3k + 2$ $\quad \therefore \; x = -7(-1) = 7$

$-3 = 3k$ $\quad z = -1$

$k = -1$

When $y = -1$

$(7, -1, -1)$

is solution.

Q5. $x = \frac{1+t}{1-t}$

$\boxed{\begin{array}{ll} u = 1+t & v = 1-t \\ u' = 1 & v' = -1 \end{array}}$

$$\frac{dx}{dt} = \frac{1.(1-t) - (1+t).-1}{(1-t)^2} = \frac{1-\cancel{t}+1+\cancel{t}}{(1-t)^2} = \frac{2}{(1-t)^2}$$

0

$$y = (1+t)(1-t)^2$$

$\boxed{\begin{array}{ll} u = 1+t & v = (1-t)^2 \\ u' = 1 & v' = 2(1-t)'.-1 \\ & \quad = -2(1-t) \end{array}}$

$$\frac{dy}{dt} = (1-t)^2 + (1+t).-2(1-t)$$
$$= (1-t)^2 - 2(1+t)(1-t)$$
$$= (1-t)[(1-t) - 2(1+t)]$$
$$= (1-t)[1-t-2-2t]$$
$$= (1-t)(-1-3t)$$
$$= -(1-t)(3t+1)$$

0

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-(1-t)(3t+1)}{\frac{2}{(1-t)^2}} = -(1-t)(3t+1) \times \frac{(1-t)^2}{2}$$

0

$$= -\frac{(1-t)^3(3t+1)}{2}$$

$$\therefore \frac{dy}{dx} = -\frac{1}{2}(3t+1)(1-t)^3$$

0

4

Where $a = 3$; $b = 1$; $k = -\frac{1}{2}$ and $l = 3$

$$\frac{dy}{dx} = -\frac{1}{2}(3t+1)(1-t)^3$$

$$= \left(-\frac{3}{2}t - \frac{1}{2}\right)(1-t)^3$$

$u = -\frac{3}{2}t - \frac{1}{2}$ $\quad v = (1-t)^3$

$u' = -\frac{3}{2}$ $\quad v' = 3(1-t)^2 \cdot -1$

$\quad = -3(1-t)^2$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = u'v + uv'$$

$$= -\frac{3}{2}(1-t)^3 - 3(1-t)^2 \times \left(-\frac{1}{2}(3t+1)\right)$$

$$= -\frac{3}{2}(1-t)^3 + \frac{3}{2}(3t+1)(1-t)^2$$

$$= \frac{3}{2}(1-t)^2\left[(3t+1) - (1-t)\right]$$

$$= \frac{3}{2}(1-t)^2(3t+1-1+t)$$

$$= \frac{3}{2}(1-t)^2(4t)$$

$$\therefore \frac{d}{dt}\left(\frac{dy}{dx}\right) = 6t(1-t)^2$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{6t(1-t)^2}{\frac{2}{(1-t)^2}} = \frac{6t(1-t)^2}{1} \times \frac{(1-t)^2}{2}$$

$$\therefore \frac{d^2y}{dx^2} = 3t(1-t)^4$$

3

Where $m = 3$ & $n = 4$

(c)

$x = \dfrac{1+t}{1-t}$

$y = (1+t)(1-t)^2$

At $t=3$

$x = \dfrac{1+3}{1-3} = \dfrac{4}{-2} = -2$

$y = (1+3)(1-3)^2$
$= (4)(-2)^2$
$= 4 \times 4$
$= 16$ $\Rightarrow (-2, 16)$ •

Gradient $m = \dfrac{dy}{dx} = -\dfrac{1}{2}(3t+1)(1-t)^3$

$= -\dfrac{1}{2}(3(3)+1)(1-(3))^3$

$= -\dfrac{1}{2} \times (10) \times (-8)$

$\therefore \quad m = 40$ •

At $(-2, 16)$, $m = 40$

Equation of tangent $(y-b) = m(x-a)$

$(y-16) = 40(x-(-2))$

$y-16 = 40(x+2)$

$y-16 = 40x+80$

$y = 40x+96$ •

3

Q6. $f(x) = x^7 + \sin(x)$

(a)

$f(-x) = (-x)^7 + \sin(-x)$

$= -x^7 - \sin(x)$

$= -(x^7 + \sin(x))$

$= -f(x)$

As $f(-x) = -f(x) \Rightarrow$ ODD FUNCTION 3

As ODD + ODD = ODD

(b)

$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{2} (x^7 + \sin(x))\,dx$

$= \left[x^8 - \cos x\right]_{-2}^{2}$

$= \left[(2)^8 - \cos(2)\right] - \left[(-2)^8 - \cos(-2)\right]$

(Even fn.)

$= (2^8 - \cos(2)) - (2^8 - \cos(2))$

$= \underline{\underline{0}}$ As cancel out 1

Q7. $\boxed{u_n = ar^{n-1}}$

$$\left.\begin{aligned} u_1 &= a \\ u_2 &= ar \end{aligned}\right\} \quad S_2 = a + ar = 30 \Rightarrow a(1+r) = 30 \quad -①$$

$$\left.\begin{aligned} u_3 &= ar^2 \\ u_4 &= ar^3 \end{aligned}\right\} \quad u_3 + u_4 = 480 \Rightarrow ar^2 + ar^3 = 480$$

So $ar^2(1+r) = 480$ –②

$$\frac{ar^2(1+r)}{a(1+r)} = \frac{480}{30}$$

$$r^2 = 16$$

$$\therefore \quad r = \pm 4$$

$\boxed{a + ar = 30 \quad -①}$

$\underline{r=4}$ $\quad a + 4a = 30$ & $\underline{r=-4}$ $\quad a - 4a = 30$ 4

$5a = 30 \qquad -3a = 30$

$a = 6 \qquad a = -10$

$\boxed{S_n = \frac{a(1-r^n)}{1-r}}$

$$S_7 = \frac{6(1-(4)^7)}{1-4} = \frac{6(1-16384)}{-3} = -2(-16383) = 32766$$

$$S_7 = \frac{-10(1-(-4)^7)}{1-(-4)} = \frac{-10(1+4^7)}{5} = -2(16385) = -32770$$

2

Method 1

$$\int_{-4}^{-2} \frac{dx}{x^2+8x+20}$$

$$= \int_0^2 \frac{1}{(u-4)^2+8(u-4)+20} \cdot du$$

$$= \int_0^2 \frac{du}{u^2-8u+16+8u-32+20}$$

$$= \int_0^2 \frac{du}{u^2+4}$$

$$= \int_0^2 \frac{du}{4\left(\frac{u^2}{4}+1\right)}$$

$$= \frac{1}{4}\int_0^2 \frac{du}{1+\left(\frac{u}{2}\right)^2}$$

$$= \frac{1}{4}\left[\frac{\tan^{-1}(u/2)}{\frac{1}{2}}\right]_0^2$$

$$= \frac{1}{2}\left(\tan^{-1}(1) - \tan^{-1}(0)\right)$$

$$= \frac{1}{2}\left(\frac{\pi}{4} - 0\right)$$

$$= \frac{\pi}{8}$$

$u = x + 4$

$\underline{x=-4}$ $\quad u = -4+4 = \underline{0}$

$\underline{x=-2}$ $\quad u = -2+4 = \underline{2}$

$u = x+4$

$\frac{du}{dx} = 1 \Rightarrow du = dx$

Method 2

Alternatively if remember rules of inverse tan when integrating

$$\int_0^2 \frac{du}{u^2+2^2}$$

$$= \left[\frac{1}{2}\tan^{-1}(u/2)\right]_0^2$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{2}{2}\right) - \frac{1}{2}\tan^{-1}(0)$$

$$= \frac{1}{2} \times \frac{\pi}{4} - 0$$

$$= \frac{\pi}{8}$$

(5)

Q8. Method 3

(If notice you can complete the square)

Can do this, but quickly becomes same problem of Inverse tan integral.

$$\int_{-4}^{-2} \frac{dx}{x^2+8x+20}$$

$$= \int_{-4}^{-2} \frac{dx}{(x+4)^2+4}$$

Let $u = x+4$

$du/dx = 1$

$du = dx$

$x=-4$: $u = -4+4 = 0$

$x=-2$: $u = -2+4 = 2$

$$= \int_{0}^{2} \frac{du}{u^2+4}$$

$$= \int_{0}^{2} \frac{du}{u^2+2^2}$$

$$= \left[\frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)\right]_0^2$$

$$= \left(\frac{1}{2}\tan^{-1}\left(\frac{2}{2}\right)\right) - \left(\frac{1}{2}\tan^{-1}(0)\right)$$

(5)

$$= \frac{1}{2} \times \frac{\pi}{4} - 0$$

$$= \frac{\pi}{8}$$

Whichever method 5 marks

1 - $\int_0^2$

2 - $\int \frac{du}{u^2+4}$ All in terms of u

3 - Inverse tan integral $\frac{1}{2}\tan^{-1}(u/2)$

4 - Subst

5 - $\pi/8$ answer.

Q9.

$$T_n = \sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)(2r+3)} \ \& \ T_n = \frac{1}{12} - \frac{1}{4(2n+1)(2n+3)}$$

Let $n=1$ $\quad T_1 = \frac{1}{(2-1)(2+1)(2+3)} = \frac{1}{1\times3\times5} = \frac{1}{15}$ on LHS

$$T_1 = \frac{1}{12} - \frac{1}{4(2+1)(2+3)} = \frac{1}{12} - \frac{1}{4\times3\times5} = \frac{1}{12} - \frac{1}{60} = \frac{5}{60} - \frac{1}{60} = \frac{4}{60} = \frac{1}{15} \text{ on RHS}$$

Thus true for $n=1$ ✓

Assume true for $n=k$

$$T_n = \sum_{r=1}^{n=k} \left(\frac{1}{(2r-1)(2r+1)(2r+3)}\right) = \frac{1}{12} - \frac{1}{4(2k+1)(2k+3)}$$

Consider for $n=k+1$

$$\sum_{r=1}^{n=k} \frac{1}{(2r-1)(2r+1)(2r+3)} + \left(\frac{1}{(2(k+1)-1)(2(k+1)+1)(2(k+1)+3)}\right) = \frac{1}{12} - \frac{1}{4(2k+1)(2k+3)} + \frac{1}{(2(k+1)-1)(2(k+1)+1)(2(k+1)+3)}$$

$$= \frac{1}{12} - \frac{1}{4(2k+1)(2k+3)} + \frac{1}{(2k+1)(2k+3)(2k+5)}$$

$$= \frac{1}{12} - \frac{(2k+5)}{4(2k+1)(2k+3)(2k+5)} + \frac{4}{4(2k+1)(2k+3)(2k+5)}$$

$$= \frac{1}{12} + \frac{4-(2k+5)}{4(2k+1)(2k+3)(2k+5)}$$

$$= \frac{1}{12} + \frac{4-2k-5}{4(2k+1)(2k+3)(2k+5)}$$

Q9.

$$= \frac{1}{12} + \frac{(-2k-1)}{4(2k+1)(2k+3)(2k+5)}$$

$$= \frac{1}{12} - \frac{\cancel{(2k+1)}}{4\cancel{(2k+1)}(2k+3)(2k+5)}$$

$$= \frac{1}{12} - \frac{1}{4(2k+3)(2k+5)}$$

$$= \frac{1}{12} - \frac{1}{4(2k+2+1)(2k+2+3)}$$

$$= \frac{1}{12} - \frac{1}{4(2(k+1)+1)(2(k+1)+3)}$$

Let $k+1=N$ (Not nec. but helps compare to original statement)

$$= \frac{1}{12} - \frac{1}{4(2N+1)(2N+3)}$$ as required

Thus as true for $n=1$, assumed true for $n=k$ and by proof of mathematical induction true for $n=k+1$. Assume true $\forall n \in N$.

$$\lim_{n\to\infty}(T_n) = \frac{1}{12} - \frac{1}{4(\infty)(\infty)} = \frac{1}{12} - 0 = \frac{1}{12}$$

6

As $n\to\infty$ limit tends to $1/12$

Q10.

$$f(x) = \frac{2^x(1-x)}{\sqrt{x}}$$

$$\ln|f(x)| = \ln\left|\frac{2^x(1-x)}{\sqrt{x}}\right|$$

$$\ln|f(x)| = \ln(2^x) + \ln(1-x) - \ln(x^{1/2})$$

$$\ln|f(x)| = x.\ln|2| + \ln(1-x) - \frac{1}{2}\ln|x|$$

$$\left(\frac{1}{f(x)}\right).f'(x) = \ln|2| + \frac{1}{(1-x)}\cdot -1 - \frac{1}{2}\cdot\frac{1}{x}$$ 4

$$f'(x) = \left(\ln|2| - \frac{1}{(1-x)} - \frac{1}{2x}\right) \times f(x)$$

$$\therefore f'(x) = \left(\ln|2| - \frac{1}{(1-x)} - \frac{1}{2x}\right) \times \left(\frac{2^x(1-x)}{\sqrt{x}}\right)$$

$$f'(\tfrac{1}{2}) = \left(\ln|2| - \frac{1}{(1-\frac{1}{2})} - \frac{1}{2(\frac{1}{2})}\right) \times \left(\frac{2^{\frac{1}{2}}(1-\frac{1}{2})}{\sqrt{1/2}}\right)$$

$$= \left(\ln|2| - \frac{1}{(\frac{1}{2})} - \frac{1}{1}\right) \times \left(\frac{\sqrt{2}\,(1/2)}{\frac{1}{\sqrt{2}}}\right)$$

$$= (\ln|2| - 2 - 1) \times (2(1/2))$$

$$= \ln|2| - 3$$

$$= \ln|2| - 3(\ln_e e)$$

$$= \ln|2| - \ln|e^3|$$ 3

$$\therefore f'(\tfrac{1}{2}) = \ln\left|\frac{2}{e^3}\right|$$

Q11. (a)

$$4z + 5w = 23 \quad \text{—} 1$$
$$2z - 3w = -5 + 22i \quad \text{—} 2$$

$$4z + 5w = 23 \quad \text{—} 1$$
$$2\times 2 \quad 4z - 6w = -10 + 44i \quad \text{—} 3$$

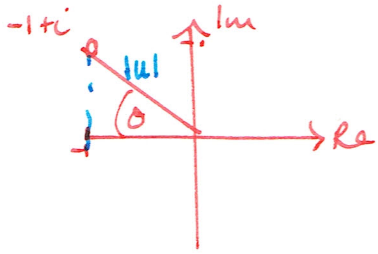
$1 - 3$
$$11w = 33 - 44i$$
$$\therefore \quad w = 3 - 4i$$

Subst w into 1

$$4z + 5w = 23$$
$$4z + 5(3-4i) = 23$$
$$4z + 15 - 20i = 23$$
$$4z = 8 + 20i$$
$$\therefore \quad z = 2 + 5i$$

3

(b)
$$u = z - \bar{w}$$
$$= (2+5i) - (3+4i)$$
$$= 2 + 5i - 3 - 4i$$
$$\therefore \quad u = -1 + i$$



$$|u| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

$$\arg(u) = \tan^{-1}\left|\frac{1}{-1}\right| = \tan^{-1}|1| = \pi/4$$

2nd Quad : $\pi - \pi/4 = 3\pi/4$

$$\therefore \quad u = r(\cos\theta + i\sin\theta)$$
$$u = \sqrt{2}\left(\cos\left(\tfrac{3\pi}{4}\right) + i\sin\left(\tfrac{3\pi}{4}\right)\right)$$

3

Q11(c) If $u = \sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$

$$u^8 = \left[\sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)\right]^8$$

$$= \left(2^{1/2}\right)^8\left(\cos\left(8 \times \frac{3\pi}{4}\right) + i\sin\left(8 \times \frac{3\pi}{4}\right)\right)$$

$$= 2^4\left(\cos(6\pi) + i\sin(6\pi)\right)$$

$$= 16\left(1 + i(0)\right)$$

$$= \underline{\underline{16}}$$

2

Q12(a) $\dfrac{1}{y^2-1} = \dfrac{1}{(y-1)(y+1)} = \dfrac{A}{(y-1)} + \dfrac{B}{(y+1)}$

$\therefore \quad A(y+1) + B(y-1) = 1$

Let $y=1$: $\quad 2A = 1$

$\therefore \underline{A = 1/2}$

Let $y=-1$: $\quad -2B = 1$

$\therefore \underline{B = -1/2}$

So $\dfrac{1}{y^2-1} = \dfrac{1/2}{(y-1)} + \dfrac{-\frac{1}{2}}{(y+1)}$

$$\frac{1}{y^2-1} = \frac{1}{2(y-1)} - \frac{1}{2(y+1)}$$

2

12(b) $\int \frac{dx}{\tan x} = \int \frac{dx}{\left(\frac{\sin x}{\cos x}\right)} = \int \frac{\cos x \, dx}{\sin x}$ [Rearrange + subst. unless can recall $\int \cot x$ by memory]

$= \int \frac{\cos x}{u} \times \frac{du}{\cos x}$

$= \int \frac{du}{u}$

Let $u = \sin x$

$du/dx = \cos x$

$\frac{du}{\cos x} = dx$

$= \ln|u| + C$

$= \ln|\sin x| + C$

(2)

(c) $2\left(\frac{dy}{dx}\right) = (y^2 - 1)\cot x$

$\frac{2}{(y^2-1)} \, dy = \cot x \, dx$

$2\left[\frac{1}{2(y-1)} - \frac{1}{2(y+1)}\right] dy = \frac{dx}{\tan x}$

$\int \left(\frac{1}{(y-1)} - \frac{1}{(y+1)}\right) dy = \int \frac{dx}{\tan x}$

$\ln|y-1| - \ln|y+1| = \ln|\sin x| + C$

$\ln\left|\frac{y-1}{y+1}\right| = \ln|\sin x| + C$

May substitute prior to rearranging to y= for ease at this stage.

$\ln\left|\frac{5-1}{5+1}\right| = \ln\left|\sin\left(\frac{3\pi}{2}\right)\right| + C$

$\ln\left|\frac{4}{6}\right| = \ln|-1| + C$

(Use Modulus.)

$\ast \ln|-1| = \ln|1| = 0$

$C = \ln|2/3| - \ln|1|$

$C = \ln|2/3|$

$$\ln\left|\frac{y-1}{y+1}\right| = \ln|\sin x| + c$$

(If $c = \ln|\frac{2}{3}|$ can now subst in)

$$\ln\left|\frac{y-1}{y+1}\right| = \ln|\sin x| + \ln|2/3|$$

$$\ln\left|\frac{y-1}{y+1}\right| = \ln\left|\frac{2}{3}\sin x\right|$$

Collect so ln |?| on each side can simplify

$$\frac{y-1}{y+1} = \frac{2}{3}\sin x$$

$$y-1 = \left(\frac{2}{3}\sin x\right)(y+1)$$

$$y-1 = \frac{2}{3}y\sin x + \frac{2}{3}\sin x$$

$$y - \frac{2}{3}y\sin x = \frac{2}{3}\sin x + 1$$

$$y\left(1-\frac{2}{3}\sin x\right) = \left(\frac{2}{3}\sin x + 1\right)$$

- Cross multiply
- Expand
- Rearrange to collect y terms
- Factorise
- express as y =

$$\therefore\ y = \frac{(1 + 2/3\sin x)}{(1 - 2/3\sin x)}$$

6

is Particular Solution