Q1.

$$\left(\begin{array}{ccc|c} 7 & 3 & 4 & 29 \\ 10 & 2 & 3 & 26 \\ 9 & 4 & 5 & 37 \end{array}\right)\begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} \rightarrow \begin{array}{c} 90r_1 \\ 63r_2 \\ 70r_3 \end{array}\left(\begin{array}{ccc|c} 630 & 270 & 360 & 2610 \\ 630 & 126 & 189 & 1638 \\ 630 & 280 & 350 & 2590 \end{array}\right)$$

$$\begin{array}{c} \frac{1}{90}r_1 \\ r_2 - r_1 \\ r_3 - r_1 \end{array}\left(\begin{array}{ccc|c} 7 & 3 & 4 & 29 \\ 0 & -144 & -171 & -972 \\ 0 & 10 & -10 & -20 \end{array}\right)\begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} \rightarrow \begin{array}{c} r_1 \\ r_2 \\ \frac{1}{10}r_3 \end{array}\left(\begin{array}{ccc|c} 7 & 3 & 4 & 29 \\ 0 & -144 & -171 & -972 \\ 0 & 1 & -1 & -2 \end{array}\right)\begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array}$$

$$\begin{array}{c} r_1 \\ r_2 \\ 144r_3 \end{array}\left(\begin{array}{ccc|c} 7 & 3 & 4 & 29 \\ 0 & -144 & -171 & -972 \\ 0 & 144 & -144 & -288 \end{array}\right)\begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} \rightarrow \begin{array}{c} r_1 \\ r_2 \\ r_2 + r_3 \end{array}\left(\begin{array}{ccc|c} 7 & 3 & 4 & 29 \\ 0 & -144 & -171 & -972 \\ 0 & 0 & -315 & -1260 \end{array}\right)\begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array}$$

$$-315z = -1260$$

$$\underline{\underline{z = 4}}$$

$$-144y - 171z = -972$$

$$144y + 171 \times (4) = +972$$

$$144y + 684 = 972$$

$$144y = 288$$

$$\underline{\underline{y = 2}}$$

$$7x + 3y + 4z = 29$$

$$7x + 3(2) + 4(4) = 29$$

$$7x + 6 + 16 = 29$$

$$7x = 7$$

$$\underline{\underline{x = 1}}$$

$$\therefore \underline{\underline{(1, 2, 4)}}$$

5

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②

Q2a) $\frac{d}{dx}(\tan^{-1}5x)^6 = \left(\frac{1}{1+(5x)^2}\cdot 5\right)\cdot 6(\tan^{-1}5x)^5$

$= \frac{30(\tan^{-1}(5x))^5}{1+25x^2}$ 3

b) $f(x) = \frac{(x^2-1)^3}{(x+1)}$

$u = (x^2-1)^3$
$u' = 3(x^2-1)^2\cdot 2x$
$= 6x(x^2-1)^2$

$v = (x+1)$
$v' = 1$

$\boxed{f'(x) = \frac{u'v - uv'}{v^2}}$

$f'(x) = \frac{6x(x^2-1)^2\cdot(x+1) - (x^2-1)^3\cdot 1}{(x+1)^2}$

$= \frac{(x^2-1)^2\left[6x(x+1) - (x^2-1)\right]}{(x+1)^2}$

$= \frac{(x^2-1)^2(6x^2+6x-x^2+1)}{(x+1)^2}$

$= \frac{(x^2-1)^2(5x^2+6x+1)}{(x+1)^2}$

4

$= \frac{(x^2-1)^2(5x+1)(x+1)}{(x+1)^2}$

$= \frac{(x^2-1)^2(5x+1)}{(x+1)}$

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3

Q3.

$$(a+bx)^7 = \sum_{r=0}^{7} \binom{7}{r} (a)^{7-r} (bx)^r$$

$$= \sum_{r=0}^{7} \binom{7}{r} (a)^{7-r} (b)^r \times x^r$$

If finding x^1

$x^r = x^1$

$r = 1$

$${}^7C_1 = \binom{7}{1} (a)^{7-1} (b)^1$$

$$= \frac{7!}{1!\,6!} \times a^6 b$$

$$= \frac{7 \times \cancel{6!}}{1 \times \cancel{6!}} \times a^6 b$$

$$= 7a^6 b$$

$\therefore\ {}^7C_1 = 218750$

If finding x^2

$x^r = x^2$

$r = 2$

$${}^7C_2 = \binom{7}{2} (a)^{7-2} (b)^2$$

$$= \frac{7!}{2!\,5!} \times a^5 \times b^2$$

$$= \frac{7 \times 6 \times \cancel{5!}}{2 \times \cancel{5!}} \times a^5 b^2$$

$$= 21a^5 b^2$$

$\therefore\ {}^7C_2 = 262500$

BOTH

COEFFS BOTH

5

$$\therefore \frac{21a^5b^2}{7a^6b} = \frac{262500}{218750}$$

$$\frac{3b}{a} = \frac{6}{5}$$

$$\therefore\ b = \frac{2}{5}a$$

$$7a^6 b = 218750$$

$$7 \times a^6 \times \left(\frac{2}{5}a\right) = 218750$$

$$\frac{14}{5}a^7 = 218750$$

$$a^7 = 78125$$

$$a = 5$$

$$\therefore\ b = \tfrac{2}{5}(5) = 2$$

Q4 $z = 4 - 3i \Rightarrow \bar{z} = 4 + 3i$

$\therefore (z - 4 + 3i) = 0$ & $(z - 4 - 3i) = 0$ are factors.

	z	-4	$+3i$
z	z^2	$-4z$	$\cancel{+3iz}$
-4	$-4z$	$+16$	$\cancel{-12i}$
$-3i$	$\cancel{-3iz}$	$\cancel{+12i}$	$-9i^2$

$(z - 4 + 3i)(z - 4 - 3i)$

$= z^2 - 8z + 16 - 9i^2$

$= z^2 - 8z + 25$

DIVIDE

$$
\begin{array}{r|l}
 & z^2 - 4z + 5 \\
\hline
z^2 - 8z + 25 & z^4 - 12z^3 + 62z^2 - 140z + 125 \\
 & z^4 - 8z^3 + 25z^2 \\
\hline
 & \quad -4z^3 + 37z^2 - 140z + 125 \\
 & \quad -4z^3 + 32z^2 - 100z \\
\hline
 & \qquad 5z^2 - 40z + 125 \\
 & \qquad 5z^2 - 40z + 125 \\
\hline
 & \qquad \_\_\_\_\_\_
\end{array}
$$

6

$\therefore z^4 - 12z^3 + 62z^2 - 140z + 125 = (z^2 - 8z + 25)(z^2 - 4z + 5)$

$\boxed{z^2 - 4z + 5 = 0}$

$z = \dfrac{4 \pm \sqrt{16 - 4 \times 1 \times 5}}{2 \times 1}$

$= \dfrac{4 \pm \sqrt{-4}}{2}$

$= \dfrac{4 \pm 2i}{2}$

$\therefore z = 2 \pm i$

So 4 solutions are:

$z = 4 \pm 3i$

& $z = 2 \pm i$

Q5.
$$\frac{2x^2+x+10}{x^3+2x^2+4x+8}$$

$$\begin{array}{r|rrrr} & 1 & 2 & 4 & 8 \\ -2 & \downarrow & -2 & 0 & -8 \\ \hline & 1 & 0 & 4 & \underline{\underline{0}} \end{array}$$

$x=-2$ is a factor as $R=0$

$\therefore\ (x+2)(x^2+4)$ factorised

$$= \frac{2x^2+x+10}{(x+2)(x^2+4)}$$

$$\frac{2x^2+x+10}{(x+2)(x^2+4)} = \frac{A}{(x+2)} + \frac{(Bx+C)}{x^2+4}$$

$$\boxed{A(x^2+4) + (Bx+C)(x+2) = 2x^2+x+10}$$

Let $x=-2$:
$$8A = 2(4)-2+10$$
$$8A = 16$$
$$\therefore\ A=2$$

Let $x=0$:
$$4A + C(2) = 10$$
$$8 + 2C = 10$$
$$2C = 2$$
$$\therefore\ C=1$$

Let $x=1$:
$$5A + (B+C)(3) = 2+1+10$$
$$10 + 3B + 3 = 13$$
$$3B = 0$$
$$\therefore\ B=0$$

3

$$\therefore\ \boxed{\frac{2x^2+x+10}{(x+2)(x^2+4)} = \frac{2}{(x+2)} + \frac{1}{(x^2+4)}}$$

Q5(b) $\displaystyle\int_0^2 \left(\frac{2x^2+x+10}{x^3+2x^2+4x+8}\right) dx$

$$= \int_0^2 \left(\frac{2}{(x+2)} + \frac{1}{(x^2+4)}\right) dx$$

$$= \left[2\ln|x+2| + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$= \left(\ln|2+2|^2 + \frac{1}{2}\tan^{-1}\left(\frac{2}{2}\right)\right) - \left(\ln|0+2|^2 + \frac{1}{2}\tan^{-1}(0)\right)$$

$$= \left(\ln|16| + \frac{1}{2}\times\left(\frac{\pi}{4}\right)\right) - \left(\ln|4| + 0\right)$$

$$= \ln|16| + \frac{\pi}{8} - \ln|4|$$

5

$$= \ln\left|\frac{16}{4}\right| + \frac{\pi}{8}$$

$$= \underline{\underline{\ln|4| + \frac{\pi}{8}}} \quad \left(\text{or } 2\ln|2| + \frac{\pi}{8}\right)$$

Q6. $x = 4\cos\theta - 5\sin\theta$ | $y = 5\cos\theta + 4\sin\theta$

$dx/d\theta = -4\sin\theta - 5\cos\theta$ | $dy/d\theta = -5\sin\theta + 4\cos\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{(-5\sin\theta + 4\cos\theta)}{(-4\sin\theta - 5\cos\theta)} = \frac{-5\sin(\pi/2) + 4\cos(\frac{\pi}{2})}{-4\sin(\pi/2) - 5\cos(\pi/2)}$$

$$\therefore \frac{dy}{dx} = \frac{-5+0}{-4-0} = \underline{\underline{5/4}}$$

5

Q7. $\int_0^{3/4} \sqrt{9-16x^2}\,dx$

$= \int_0^{3/4} \sqrt{9-(4x)^2}\,dx$

$= \int_0^{\pi/2} \sqrt{9-(3\sin\theta)^2} \times \frac{3}{4}\cos\theta\,d\theta$

$= \int_0^{\pi/2} \sqrt{9-9\sin^2\theta} \times \frac{3}{4}\cos\theta\,d\theta$

$= \frac{3}{4}\int_0^{\pi/2} \sqrt{9(1-\sin^2\theta)}\cos\theta\,d\theta$

$= \frac{3}{4}\int_0^{\pi/2} 3\sqrt{\cos^2\theta} \times \cos\theta\,d\theta$

$= \frac{9}{4}\int_0^{\pi/2} \cos^2\theta\,d\theta$

$= \frac{9}{4}\int_0^{\pi/2} \left(\frac{\cos 2\theta}{2}+\frac{1}{2}\right)d\theta$

$= \frac{9}{8}\int_0^{\pi/2} (\cos 2\theta + 1)\,d\theta$

$= \frac{9}{8}\left[\frac{\sin 2\theta}{2} + \theta\right]_0^{\pi/2}$

$= \frac{9}{8}\left(\left(\frac{\sin\left(2\left(\frac{\pi}{2}\right)\right)}{2} + \frac{\pi}{2}\right) - \left(\frac{\sin 0}{2} + 0\right)\right)$

$= \frac{9}{8}\left(0 + \pi/2 - 0\right)$

$= 9\pi/16$

$4x = 3\sin\theta$

$x = \frac{3}{4}\sin\theta$

$\frac{dx}{d\theta} = \frac{3}{4}\cos\theta$

$\therefore dx = 3/4 \cos\theta\,d\theta$

$x=0$ $4(0) = 3\sin\theta$

$3\sin\theta = 0$

$\sin\theta = 0$

$\theta = 0$

$x = \frac{3}{4}$ $4\left(\frac{3}{4}\right) = 3\sin\theta$

$3 = 3\sin\theta$

$\therefore \sin\theta = 1$

$\theta = \frac{\pi}{2}$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta + 1 = 2\cos^2\theta$

$\frac{\cos 2\theta + 1}{2} = \cos^2\theta$

$\therefore \cos^2\theta = \left(\frac{\cos 2\theta}{2} + \frac{1}{2}\right)$

7

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Q8. (a) $f(x) = \dfrac{6}{(x+1)(x-3)}$

Vertical Asymptotes

$x = -1$ & $x = 3$

As $x \to \pm\infty$ $y \to 0$ $\Rightarrow$ Horizontal Asymptote

$y = 0$

3

(b) $f(x) = \dfrac{6}{(x^2 - 3x + x - 3)} = \dfrac{6}{(x^2 - 2x - 3)} = 6(x^2 - 2x - 3)^{-1}$

$f'(x) = -6(x^2 - 2x - 3)^{-2} \cdot (2x - 2)$

$= \dfrac{-6(2x-2)}{(x^2 - 2x - 3)^2}$

Stat Pt at $f'(x) = 0$ $\qquad \dfrac{-6(2x-2)}{(x^2-2x-3)^2} = 0$

$-6(2x-2) = 0$

$2x - 2 = 0$

$2x = 2$

$\therefore x = 1$

y-coord at $x = 1$: $y = \dfrac{6}{(1+1)(1-3)} = \dfrac{6}{2x-2} = \dfrac{6}{-4} = -\dfrac{3}{2}$

$\therefore$ Stat Pt $(1, -3/2)$ exists

(To find nature will need to use quotient rule $\rightarrow$ P.T.O.)

Q8 (b) Continued

$$f'(x) = \frac{-12x-12}{(x^2-2x-3)^2}$$

$u = -12x-12 \qquad v = (x^2-2x-3)^2$

$u' = -12 \qquad v' = 2(2x-2) \times (x^2-2x-3)$

$$\boxed{f''(x) = \frac{u'v-uv'}{v^2}}$$

$$f''(x) = \frac{-12(x^2-2x-3)^2 - (-12x-12)(2(2x-2)(x^2-2x-3))}{[(x^2-2x-3)^2]^2}$$

$$= \frac{-12(x^2-2x-3)^2 + (12x+12)(4x-4)(x^2-2x-3)}{(x^2-2x-3)^4}$$

$$= \frac{(x^2-2x-3)\left[-12(x^2-2x-3) + (12x+12)(4x-4)\right]}{(x^2-2x-3)^4}$$

$$= \frac{-12x^2+24x+36+48x^2\cancel{-48x}\cancel{+48x}-48}{(x^2-2x-3)^3}$$

$$= \frac{36x^2+24x-12}{(x^2-2x-3)^3}$$

5

$$= \frac{12(3x^2+2x-1)}{[(x+1)(x-3)]^3}$$

$$= \frac{12(3x-1)(x+1)}{(x+1)^3(x-3)^3}$$

$$= \frac{12(3x-1)}{(x+1)^2(x-3)^3}$$

$$f''(1) = \frac{12(2)}{(2)^2(-2)^3} = \frac{24}{-32} < 0 \Rightarrow \cap \text{ Max TPt } (1, -3/2)$$

Q8 (c) $\boxed{y=0, \quad x=-1, \quad x=3 \qquad \text{Max}\,(1,-3/2)}$

$\underline{x=0}$ $\quad y = \frac{6}{1 \times -3} = \underline{\underline{-2}} \Rightarrow (0,-2)$ is y-intercept.

& $\underline{y=0}$ $\quad 0 \neq 6 \Rightarrow$ Doesn't cut x-axis

---

Asymptotes & Extremes

$x \to -1^+ \quad y \to \infty^- \qquad x \to 3^+ \quad y \to \infty^+$

$x \to -1^- \quad y \to \infty^+ \qquad x \to 3^- \quad y \to \infty^-$

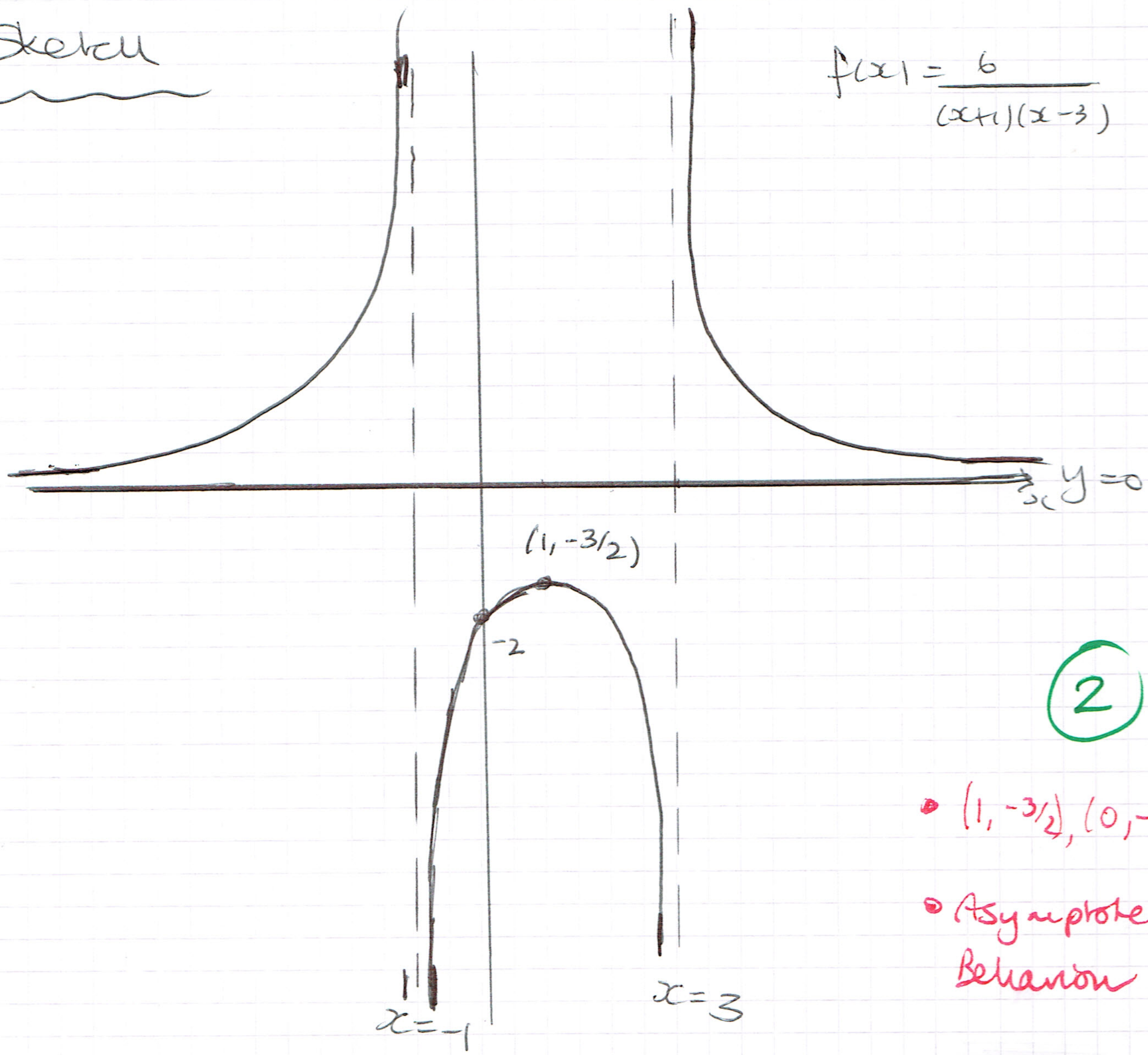
$x \to \infty^+ \quad y \to 0^+$

$x \to \infty^- \quad y \to 0^+$

$$\boxed{f(x) = \frac{6}{(x+1)(x-3)}}$$

(c) Sketch

$f(x) = \frac{6}{(x+1)(x-3)}$



2

- $(1,-3/2), (0,-2)$
- Asymptotes Behaviour

Q9. $y = \sqrt{2+\sin x} = (2+\sin x)^{1/2}$

$$\boxed{V = \int \pi y^2 \, dx}$$

$$\text{Volume} = \int_0^{3\pi/4} \pi\left(\sqrt{2+\sin x}\right)^2 dx$$

$$= \pi \int_0^{3\pi/4} (2+\sin x)\, dx$$

$$= \pi \left[2x + (-\cos x) \right]_0^{\frac{3\pi}{4}}$$

$$= \pi \left[\left(2\left(\tfrac{3\pi}{4}\right) - \cos\left(\tfrac{3\pi}{4}\right)\right) - (0 + -\cos 0) \right]$$

$$= \pi \left[\frac{3\pi}{2} - \left(-\frac{1}{\sqrt{2}}\right) - 0 + 1 \right]$$

$$= \pi \left(\frac{3\pi}{2} + \frac{1}{\sqrt{2}} + 1 \right)$$

May present with common Denom but not nec.

$$= \pi \left(\frac{3\pi}{2} + \frac{\sqrt{2}}{2} + \frac{2}{2} \right)$$

$$= \frac{\pi}{2} \left(3\pi + 2 + \sqrt{2} \right)$$

OR $20 \cdot 2$ units^3

4

Q10. $3 + xy = 2x^2$ • Method.

$$0 + x\frac{dy}{dx} + y = 4x$$

$$x\frac{dy}{dx} = 4x - y$$

$$\frac{dy}{dx} = \frac{4x-y}{x} \quad \left(= 4 - \frac{y}{x} \text{ if preferred}\right)$$

$$\frac{d^2y}{dx^2} = \frac{u'v - uv'}{v^2}$$

$u = 4x - y$ $\quad v = x$

$u' = 4 - \frac{dy}{dx}$ $\quad v' = 1$

$= 4 - \left(\frac{4x-y}{x}\right)$ SUBST.

$= 4 - 4 + \frac{y}{x}$

$\therefore u' = \frac{y}{x}$

$$= \frac{\left(\frac{y}{x}\right) \times x - (4x-y).1}{x^2}$$

$$= \frac{y - (4x-y)}{x^2}$$

$$= \frac{y - 4x + y}{x^2}$$

$$\frac{d^2y}{dx^2} = \frac{2y - 4x}{x^2}$$

$$\left(\text{or } \frac{d^2y}{dx^2} = \frac{2(y-2x)}{x^2}\right)$$

5

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Q11. $(x+1), (3x+1), (6x-2)$

$d = (3x+1) - (x+1)$

$= 3x+1-x-1$

$\therefore d = 2x$

$d = (6x-2) - (3x+1)$

$= 6x-2-3x-1$

$\therefore d = 3x-3$

If $d = 2x$ & $d = 3x-3$

$2x = 3x-3$

$-x = -3$

$\therefore x = 3$

If $a = x+1$

then $a = 3+1 = 4$

If $d = 2x$

$d = 2(3) = 6$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$= \frac{n}{2}[2\times 4 + 6\times(n-1)]$

$= \frac{n}{2}[8+6n-6]$

$= \frac{n}{2}(2+6n)$

$S_n = 3n^2+n$

4

If $n=5$; $S_5 = 3\times(5)^2+(5) = 80 < 100$ x

$n=6$: $S_6 = 3\times(6)^2+(6) = 108+6 = 114 > 100$ ✓

$\therefore S_n = 3n^2+n > 100$

when $n=6$

Q11. (b) $\boxed{a, ar, ar^2, ar^3, \ldots ar^{n-1}}$

$a + ar + ar^2 = 315 \Rightarrow a(1+r+r^2) = 315$ — (1)

$ar^4 + ar^5 + ar^6 = 80640 \Rightarrow ar^4(1+r+r^2) = 80640$ — (2)

$$\therefore \frac{\cancel{a}r^4\cancel{(1+r+r^2)}}{\cancel{a}\cancel{(1+r+r^2)}} = \frac{80640}{315}$$

$$r^4 = 256$$

$$r = \sqrt[4]{256}$$

$$\therefore r = 4$$

Take (1) $a(1+r+r^2) = 315$

$$a(1+4+4^2) = 315$$

$$21a = 315$$

$$\therefore a = 15$$

6

$\therefore U_n = ar^{n-1}$

$$\boxed{U_n = 15(4)^{n-1}}$$

$\therefore$ Sequence 15, 60, 240, 960, 3840, 15360, ...