Q1.

$$\binom{n}{0}\binom{n}{1}\binom{n}{2}\binom{n}{3}\binom{n}{4}\binom{n}{5}\binom{n}{6}\binom{n}{7}\binom{n}{8}\binom{n}{9}\binom{n}{10}$$

11 parts ⇒ 6th is middle term as shall be term in centre of expansion

$$\sum_{r=0}^{n}\binom{n}{r}(x)^{n-r}(y)^{r} \Rightarrow \left(2x-\frac{1}{x}\right)^{10} = \sum_{r=0}^{10}\binom{10}{r}(2x)^{10-r}\left(\frac{-1}{x}\right)^{r}$$

so if looking at $\binom{10}{5}$ ⇒ $r=5$ will find middle term

$$\binom{10}{5}(2x)^{10-5}\left(-\tfrac{1}{x}\right)^{5}$$

$$= \binom{10}{5}(2)^5(-1)^5 \times (x)^5(x^{-1})^5$$

$$= \frac{10!}{5!\,5!} \times 32 \times -1 \times \left(x^5 \times x^{-5}\right)$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5! \times (5\times4\times3\times2\times1)} \times -32 \times x^0$$

(3)

$$= 252 \times -32$$

$$= \underline{\underline{-8064}}$$

so middle term is independent of x.

Q2.

$x = \frac{t-1}{t+1}$

$u = t-1$, $u' = 1$, $v = t+1$, $v' = 1$

$\frac{dx}{dt} = \frac{(t+1)-(t-1)}{(t+1)^2}$

$= \frac{t+1-t+1}{(t+1)^2}$

$= \frac{2}{(t+1)^2}$

$y = \frac{t+1}{t-1}$

$u = t+1$, $u' = 1$, $v = t-1$, $v' = 1$

$\frac{dy}{dt} = \frac{(t-1)-(t+1)}{(t-1)^2}$

$= \frac{t-1-t-1}{(t-1)^2}$

$= \frac{-2}{(t-1)^2}$

(3)

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2}{(t-1)^2} \div \frac{2}{(t+1)^2} = \frac{-2}{(t-1)^2} \times \frac{(t+1)^2}{2} = \frac{-(t+1)^2}{(t-1)^2}$

b) $t = -2$ $\quad x = \frac{-2-1}{-2+1} = \frac{-3}{-1} = +3$ & $y = \frac{-2+1}{-2-1} = \frac{-1}{-3} = \frac{1}{3}$

$(m =) \frac{dy}{dx} = \frac{-(-2+1)^2}{(-2-1)^2} = \frac{-(-1)^2}{(-3)^2} = \frac{-1}{9}$ $\quad \therefore m = \frac{-1}{9}$ & $(3, \frac{1}{3})$

$(y - \frac{1}{3}) = \frac{-1}{9}(x-3)$

(3)

$9y - 3 = -x + 3$

$\therefore$ $x + 9y - 6 = 0$ is equation of tangent

OR $x + 9y = 6$ As required

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Q3.

$$\left(\begin{array}{ccc|c} 1 & -1 & -3 & 3 \\ 1 & 2 & -4 & 7 \\ 2 & -1 & 0 & 1 \end{array}\right)\begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} \rightarrow \begin{array}{c} 2r_1 \\ 2r_2 \\ r_3 \end{array}\left(\begin{array}{ccc|c} 2 & -2 & -6 & 6 \\ 2 & 4 & -8 & 14 \\ 2 & -1 & 0 & 1 \end{array}\right)\begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array}$$

$$\begin{array}{c} r_1 \\ r_2 - r_1 \\ r_3 - r_1 \end{array}\left(\begin{array}{ccc|c} 2 & -2 & -6 & 6 \\ 0 & 6 & -2 & 8 \\ 0 & 1 & 6 & -5 \end{array}\right)\begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} \rightarrow \begin{array}{c} \frac{1}{2}r_1 \\ \frac{1}{2}r_2 \\ 3r_3 \end{array}\left(\begin{array}{ccc|c} 1 & -1 & -3 & 3 \\ 0 & 3 & -1 & 4 \\ 0 & 3 & 18 & -15 \end{array}\right)\begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array}$$

$$\begin{array}{c} r_1 \\ r_2 \\ r_3 - r_2 \end{array}\left(\begin{array}{ccc|c} 1 & -1 & -3 & 3 \\ 0 & 3 & -1 & 4 \\ 0 & 0 & 19 & -19 \end{array}\right)\begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array}$$

$\therefore$ $19z = -19$; $3y - z = 4$; $x - y - 3z = 3$

$z = -1$

$3y = 4 + (-1)$

$3y = 3$

$y = 1$

$x = y + 3z + 3$

$x = 1 - 3 + 3$

$x = 1$

$\therefore$ $(1, 1, -1)$ is solution

5

Q4. $f(x) = x\tan^{-1}(x) - \ln(1+x^2)^{1/2}$

$$f(x) = x\tan^{-1}(x) - \frac{1}{2}(\ln(1+x^2))$$

$$f'(x) = \left(\tan^{-1}(x) + \frac{x}{1+x^2}\right) - \frac{1}{2}\left(\frac{1}{1+x^2}\right)\cdot 2x$$

$$= \tan^{-1}(x) + \frac{x}{1+x^2} - \frac{x}{1+x^2}$$

$$\therefore f'(x) = \tan^{-1}(x)$$

(3)

$(x\tan^{-1}(x))$
$u = x$
$u' = 1$
$v = \tan^{-1}(x)$
$v' = \frac{1}{1+x^2}$

---

b) $\ln(y) = \ln\left(\frac{(x+1)^{1/2}}{(x-1)^{1/2}}\right)$

$$\ln|y| = \ln|x+1|^{1/2} - \ln|x-1|^{1/2}$$

$$\ln|y| = \frac{1}{2}\ln(x+1) - \frac{1}{2}\ln|x-1|$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2(x+1)} - \frac{1}{2(x-1)}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x+1} - \frac{1}{x-1}\right)\cdot y$$

$$= \frac{1}{2}\left(\frac{(x-1)-(x+1)}{x^2-1}\right)\cdot y$$

$$= \frac{1}{2}\left(\frac{x-1-x-1}{x^2-1}\right)\cdot y$$

$$= \frac{1}{2}\left(\frac{-2}{x^2-1}\right)\cdot y$$

$$\therefore \frac{dy}{dx} = \left(\frac{1}{1-x^2}\right)\times y$$

As required.

(5)

Q5. $(2-4i)^4 = 16 - 128i + 384i^2 - 512i^3 + 256i^4$

$= 16 - 128i - 384 + 512i + 256$

$= (16 - 384 + 256) + i(512 - 128)$

$\therefore \quad z^4 = -112 + 384i$

$z = 2 - 4i \Rightarrow z^2 = (2-4i)^2$

$= 4 - 16i + 16i^2$

$= 4 - 16i - 16$

$\therefore z^2 = -12 - 16i$

& $z^3 = z \times z^2$

$= (2-4i)(-12-16i)$

$= -24 - 32i + 48i + 64i^2$

$= -24 - 64 + 48i - 32i$

$\therefore z^3 = -88 + 16i$

$z^4 - 10z^3 + 41z^2 - 108z - 60 = 0$

$(-112+384i) - 10(-88+16i) + 41(-12-16i) - 108(2-4i) - 60 = 0$

$-112 + 384i + 880 - 160i - 492 - 656i - 216 + 432i - 60 = 0$

$(-112 + 880 - 492 - 216 - 60) + i(384 - 160 - 656 + 432) = 0$

$(0) + i(0) = 0$

$0 = 0$

LHS = RHS ✓

(3)

Thus $z = 2-4i$ must be a factor.

Q5(b)

$z = 2-4i \Rightarrow \bar{z} = 2+4i$ is conjugate

factors are therefore: $(z-2+4i)=0$ & $(z-2-4i)=0$

Multiplying: $(z-2+4i)(z-2-4i)$

	z	-2	$+4i$
z	z^2	$-2z$	~~$+4iz$~~
-2	$-2z$	$+4$	~~$-8i$~~
$-4i$	~~$-4iz$~~	~~$+8i$~~	$-16i^2$

$= z^2 - 4z + 4 - 16i^2$

$= z^2 - 4z + 4 + 16$

$= z^2 - 4z + 20$

Dividing into Quartic gives remaining factors to solve

$z^2 - 6z - 3$

$z^2 - 4z + 20 \,\overline{)\, z^4 - 10z^3 + 41z^2 - 108z - 60}$

$z^4 - 4z^3 + 20z^2$

$-6z^3 + 21z^2 - 108z - 60$

$-6z^3 + 24z^2 - 120z$

$-3z^2 + 12z - 60$

$-3z^2 + 12z - 60$

$$z = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{6 \pm \sqrt{36-4\times1\times-3}}{2\times1}$$

$$= \frac{6 \pm \sqrt{48}}{2}$$

$$= \frac{6 \pm 4\sqrt{3}}{2}$$

$$z = 3 \pm 2\sqrt{3}$$

4 Solutions are:

$\therefore z_1 = 2-4i$

$z_2 = 2+4i$

$z_3 = 3+2\sqrt{3}$

$z_4 = 3-2\sqrt{3}$

5

Q6. $\int_0^1 \tan^{-1}(x)\,dx = \int_0^1 1 \cdot \tan^{-1}(x)\,dx$

$$\int u'v = uv - \int uv' \quad \text{Integ by Parts}$$

Let $u = x$ $v = \tan^{-1}(x)$

$u' = 1$ $v' = \frac{1}{1+x^2}$

$$\int_0^1 1 \cdot \tan^{-1}(x)\,dx$$

$$= \left[x\tan^{-1}(x)\right]_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx$$

Let $u = 1+x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$x=0$ $u = 1+0^2 = 1$

$x=1$ $u = 1+1^2 = 2$

$$= \left[x\tan^{-1}(x)\right]_0^1 - \left[\int_1^2 \frac{x}{u} \cdot \frac{du}{2x}\right]$$

$$= \left[x\tan^{-1}(x)\right]_0^1 - \int_1^2 \frac{du}{2u}$$

$$= \left[x\tan^{-1}(x)\right]_0^1 - \frac{1}{2}\int_1^2 \frac{du}{u}$$

$$= \left[x\tan^{-1}(x)\right]_0^1 - \frac{1}{2}\left[\ln|u|\right]_1^2$$

$$= \left(1 \cdot \tan^{-1}(1) - 0\right) - \frac{1}{2}\left(\ln|2| - \ln|1|\right)$$

$$= \tan^{-1}(1) - \frac{1}{2}\ln|2|$$

$$= \frac{\pi}{4} - \ln|2|^{1/2}$$

$$= \frac{\pi}{4} - \ln|\sqrt{2}|$$

As Required.

(4)

Q7. $\displaystyle\int_{\frac{5}{2}}^{5} \frac{2x}{\sqrt{100-4x^2}}\,dx$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{2(5\sin\theta).5\cos\theta d\theta}{\sqrt{100-4(5\sin\theta)^2}}$$

$$= \int_{\pi/6}^{\frac{\pi}{2}} \frac{50\sin\theta\cos\theta d\theta}{\sqrt{100-100\sin^2\theta}}$$

$$= 50\int_{\pi/6}^{\pi/2} \frac{\sin\theta\cos\theta d\theta}{\sqrt{(100(1-\sin^2\theta))}}$$

$$= \frac{50}{10}\int_{\pi/6}^{\pi/2} \frac{\sin\theta\cos\theta d\theta}{\sqrt{\cos^2\theta}}$$

$$= 5\int_{\pi/6}^{\pi/2} \sin\theta d\theta$$

$$= 5\Big[-\cos\theta\Big]_{\pi/6}^{\pi/2}$$

$$= 5\left(\left(-\cos\frac{\pi}{2}\right)-\left(-\cos\frac{\pi}{6}\right)\right)$$

$$= 5\left(0+\frac{\sqrt{3}}{2}\right)$$

$$= \frac{5\sqrt{3}}{2}$$

If $x = 5\sin\theta$ | $x = 5\sin\theta$

$5 = 5\sin\theta$ | $\frac{5}{2} = 5\sin\theta$

$\sin\theta = 1$ | $\sin\theta = 1/2$

$\therefore \theta = \pi/2$ | $\theta = \frac{\pi}{6}$

$x = 5\sin\theta$

$\frac{dx}{d\theta} = 5\cos\theta$

$dx = 5\cos\theta d\theta$

Recall

$\sin^2\theta + \cos^2\theta = 1$

So $\cos^2\theta = 1-\sin^2\theta$

& $\sqrt{\cos^2\theta} = \cos\theta$

(5)

Q8. $\frac{2x^2-4}{x^2-1}$

$$\begin{array}{r} 2 \\ x^2-1 \overline{\big)\, 2x^2 \quad -4} \\ 2x^2 \quad -2 \\ \hline -2 \end{array}$$

$\Rightarrow 2 + \frac{-2}{(x^2-1)}$

$= 2 + \frac{-2}{(x-1)(x+1)}$

$\frac{-2}{x^2-1} = \frac{A}{(x-1)} + \frac{B}{(x+1)}$

$A(x+1) + B(x-1) = -2$

Let $x=-1$: $-2B = -2$ $\therefore B = 1$

Let $x=+1$: $2A = -2$ $\therefore A = -1$

So $\frac{2x^2-4}{x^2-1} = 2 + \frac{-1}{(x-1)} + \frac{1}{(x+1)}$

where $A = 2$, $B = -1$, $C = 1$ in form required

3

(b) Vertical Asymptotes at $x=1$ & $x=-1$

(i) Horizontal Asymptote at $y=2$

2

(ii) $f(x) = 2 - (x-1)^{-1} + (x+1)^{-1}$

$f'(x) = (x-1)^{-2} - (x+1)^{-2} = 0$

$\frac{1}{(x-1)^2} - \frac{1}{(x+1)^2} = 0$

$\frac{1}{(x-1)^2} = \frac{1}{(x+1)^2}$

$(x+1)^2 = (x-1)^2$

Q8 (b) (ii)

$f'(x) = 0$

$$(x+1)^2 = (x-1)^2$$

$$x^2 + 2x + 1 = x^2 - 2x + 1$$

$$4x = 0$$

$$x = 0$$

At $x=0$: $y = \frac{0-4}{0-1} = 4$ $\therefore$ Stat Pt at $(0,4)$

$$f'(x) = (x-1)^{-2} - (x+1)^{-2}$$

$$f''(x) = -2(x-1)^{-3} + 2(x+1)^{-3}$$

$$= \frac{-2}{(x-1)^3} + \frac{2}{(x+1)^3}$$

$$= 2\left(\frac{1}{(x+1)^3} - \frac{1}{(x-1)^3}\right)$$

$x=0$: $f''(0) = 2\left(\frac{1}{(1)^3} - \frac{1}{(-1)^3}\right) = 2(1+1) = 4 > 0$

$\therefore$ $f''(0) > 0$ U Min TPT $(0,4)$

(4)

Cuts Axes Min TPt at $(0,4)$

If $y=0$

$$0 = \frac{2x^2-4}{x^2-1}$$

$$0 = 2x^2 - 4$$

$$x^2 - 2 = 0$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

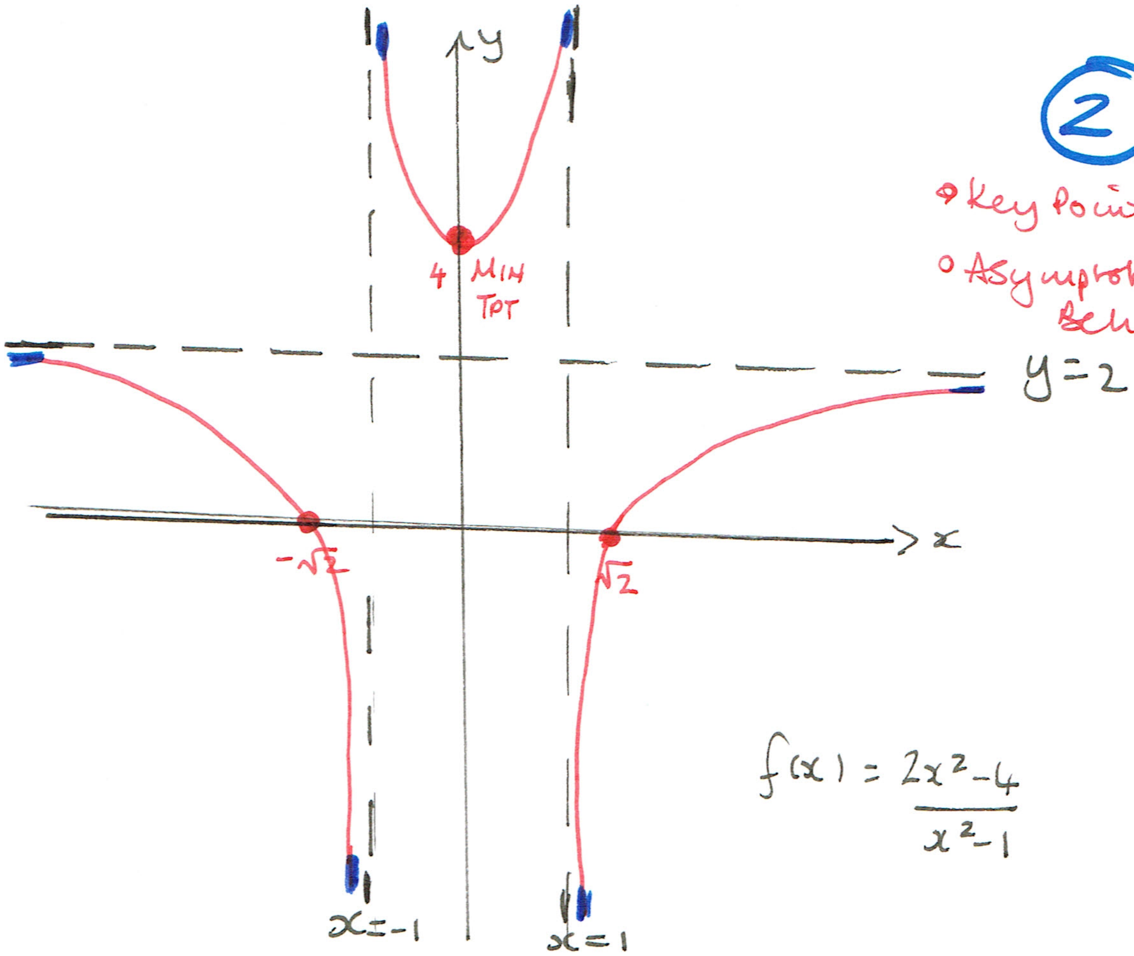
$\therefore$ Also cuts axis at $(\sqrt{2},0)$ & $(-\sqrt{2},0)$

Min (0,4)

Cut $(\sqrt{2},0)$ & $(-\sqrt{2},0)$

$$2 + \frac{-1}{x-1} + \frac{1}{x+1}$$

Asymptotes	$x=1$	$x=-1$	
$x \to 1^+$	$y \to \infty^-$	$x \to -1^+$	$y \to \infty^+$
$x \to 1^-$	$y \to \infty^+$	$x \to -1^-$	$y \to \infty^-$
As $x \to \pm\infty$			
$x \to \infty$	$y \to 2^-$		
$x \to -\infty$	$y \to 2^-$		



(2)

- Key Points
- Asymptote Behaviour

$$f(x) = \frac{2x^2-4}{x^2-1}$$

Q9 a) $\log_a x, \log_a xy, \log_a xy^2, \ldots$

$u_1 = a = \log_a x$

$d = \log_a xy - \log_a x$

$= \log_a \frac{xy}{x}$

$\therefore d = \log_a y$

OR

$d = \log_a xy^2 - \log_a xy$

$= \log_a \frac{xy^2}{xy}$

$d = \log_a y$

$u_n = a + (n-1)d$

$u_n = \log_a x + (n-1) \times \log_a y$

$u_n = \log_a x + \log_a y^{(n-1)}$

$u_n = \log_a xy^{n-1}$

(3)

(b) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_7 = \frac{7}{2}\left[2(\log_3 24) + (7-1)\log_3 \frac{1}{2}\right]$

$= 7\left(\log_3 24 + 3\log_3 \frac{1}{2}\right)$

$= 7\left(\log_3 24 + \log_3 \left(\frac{1}{2}\right)^3\right)$

$= 7\left(\log_3 \left(24 \times \frac{1}{8}\right)\right)$

$= 7\log_3(3)$

$\therefore S_7 = 7$

$a = 3 ; x = 24$

$a = \log_a x = \log_3 24$

$a = 3 ; y = \frac{1}{2}$

$d = \log_a y$

$d = \log_3 \frac{1}{2}$

(2)

'$\sqrt{2}$ is not a rational number'

Q10.

Assume statement is false and $\sqrt{2}$ is RATIONAL

$\Rightarrow \sqrt{2} = p/q$ where p & q are positive integers and do not have a common divisor.

By Squaring

$$(\sqrt{2})^2 = (p/q)^2$$

$$2 = \frac{p^2}{q^2}$$

then $p^2 = 2q^2 \Rightarrow p^2$ is even (*)

If p^2 is even $\Rightarrow p$ must be even

So let $p = 2m$ where m is a positive integer.

Then if $p^2 = 2q^2$ from (*)

$$(2m)^2 = 2q^2$$

$$4m^2 = 2q^2$$

5

$\therefore \quad q^2 = 2m^2 \Rightarrow q^2$ is even

$\Rightarrow q$ must be even too.

However we stated no common divisor exists, but p & q are both even $\Rightarrow$ common multiple of 2.

$\Rightarrow$ CONTRADICTION! Thus $\sqrt{2}$ is NOT a rational number.

Q11. $x^2 - xy - 5y^2 = 1$

$\underline{y=1}$

$x^2 - x - 5 = 1$

$x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

$x = -2$ & $x = 3$

∴ When $y = 1$ curve meets at $(-2, 1)$ & $(3, 1)$

3

$x^2 - xy - 5y^2 = 1$

$$2x - y - x\frac{dy}{dx} - 10y\frac{dy}{dx} = 0$$ 1

$$2x - y = x\frac{dy}{dx} + 10y\frac{dy}{dx}$$

$$2x - y = (x + 10y)\frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{2x - y}{x + 10y}$$ 1

3

At $(-2, 1)$

$$m = \frac{dy}{dx} = \frac{2(-2) - (1)}{(-2) + 10(1)}$$

$$= \frac{-4 - 1}{-2 + 10}$$

$$\therefore m = -5/8$$

At $(3, 1)$

$$m = \frac{dy}{dx} = \frac{2(3) - (1)}{(3) + 10(1)}$$

$$= \frac{6 - 1}{3 + 10}$$

$$\therefore m = \frac{5}{13}$$ 1

Q12 $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$ & $\frac{dr}{dt} = 0.1$ cm/s , $r = 10$ cm.

(a) $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} = 4\pi r^2 \times 0.1 = 0.4\pi r^2$

At $r = 10$cm $\frac{dV}{dt} = 0.4 \times \pi \times 10^2$

$= \underline{\underline{40\pi}}$

(4)

(b) $SA = 4\pi r^2$

$\frac{d(SA)}{dr} = 8\pi r$ & $\frac{d(SA)}{dt} = 8\pi$ cm²/s

$\frac{d(SA)}{dr} = \frac{d(SA)}{dt} \times \frac{dt}{dr}$

So $\frac{d(SA)}{dr} = \frac{\frac{d(SA)}{dt}}{dr/dt}$

(4)

$\frac{dr}{dt} = \frac{\frac{d(SA)}{dt}}{\frac{d(SA)}{dr}} = \frac{8\pi}{8\pi r} = \frac{1}{r} = \frac{1}{25}$ cm/s. (When $r = 25$cm)

Q13. (a)

$$\frac{dP}{dt} = -kP$$

$$\frac{dP}{P} = -k\,dt$$

$$\int \frac{dP}{P} = \int -k\,dt$$

$$\ln|P| = -kt + C$$

$$P = e^{-kt+c} = e^{kt}.e^{c}$$

- Separate Variables
- Integrate
- Use indice rules

At $t=0$; Initial Population $= P_0$

$$\therefore\ P = e^{-kt}.e^{c}$$

$\underline{t=0}$ $\quad P_0 = e^{0}.e^{c}$

$$\therefore\ P_0 = e^{c}$$

So $\quad P = e^{kt}.e^{c}$

is $\quad P = P_0 e^{-kt}$

(4)

(b)

$t = 100$
$P = \frac{2}{3}P_0$

$$\frac{2}{3}P_0 = P_0 e^{-100k}$$

$$\frac{2}{3} = e^{-100k}$$

$$\ln|2/3| = -100k$$

$$k = \frac{\ln|2/3|}{-100}$$

$$k = 0.00405 \text{ (to 3sf)}$$

Half Population

$$\frac{1}{2}P_0 = P_0 e^{-0.00405t}$$

$$0.5 = e^{-0.00405t}$$

$$\ln|0.5| = -0.00405t$$

$$\therefore t = \frac{\ln|0.5|}{-0.00405}$$

$$t = 171.147\ldots$$

$t = 171.1$ mins

($\sim$ 2hrs 51mins)

(5)

Q14.

(a) Area $= \int_0^1 \left(\frac{x}{x+2}\right) dx$

$= \int_0^1 \left(\frac{x+2-2}{x+2}\right) dx$

$= \int_0^1 \left(\frac{x+2}{x+2} - \frac{2}{x+2}\right) dx$

$= \int_0^1 \left(1 - \frac{2}{x+2}\right) dx$

$= \left[x - 2\ln|x+2|\right]_0^1$

$= (1 - 2\ln|3|) - (0 - 2\ln|2|)$

$= 1 - \ln|3|^2 + \ln|2|^2$

$= 1 - \ln|9| + \ln|4|$

$= 1 + \ln\left|\frac{4}{9}\right|$

So Shaded Area $= (1\times1) - (1 + \ln|4/9|)$

$= 1 - 1 - \ln|4/9|$

$= -\ln|4/9|$

$= \ln|4/9|^{-1}$

$= \ln\left|\frac{1}{4/9}\right|$

$= \ln|9/4|$

(5)

Q14 (b)

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(18)

$$V = \int \pi y^2 dx \quad \text{or} \quad \int \pi x^2 dy$$

As x-axis use $\int \pi y^2 dx$

$$V = \int_a^b \pi \times (y^2)\, dx$$

$$V = \int_0^1 \pi \left(\frac{x}{x+2}\right)^2 dx$$

$$= \pi \int_0^1 \left(\frac{x}{x+2}\right)^2 dx$$

$$= \pi \int_0^1 \left(1 - \frac{2}{x+2}\right)^2 dx$$

$$= \pi \int_0^1 \left(1 - \frac{4}{x+2} + \frac{4}{(x+2)^2}\right) dx$$

5

$$= \pi \int_0^1 \left(1 - \frac{4}{x+2} + 4(x+2)^{-2}\right) dx$$

$$= \pi \left[x - 4\ln|x+2| + \frac{4(x+2)^{-1}}{-1} \right]_0^1$$

$$= \pi \left[\left(1 - 4\ln|3| - \frac{4}{(1+2)^1}\right) - \left(0 - 4\ln|2| - \frac{4}{(0+2)}\right) \right]$$

$$= \pi \left(1 - 4\ln|3| - \frac{4}{3} - 0 + 4\ln|2| + \frac{4}{2}\right)$$

$$= \pi \left(1 + 2 - \frac{4}{3} + \ln|2|^4 - \ln|3|^4\right)$$

$$= \pi \left(5/3 + \ln\left|\frac{16}{81}\right| \right) \text{ units}^3 \quad (\text{or } 0.141)$$

450, 360, 288, . . .

Q15.

$r = \frac{360}{450} = \frac{4}{5} = \frac{288}{360}$

& $a = 450$

$U_n = ar^{n-1}$

$U_n = 450 \times \left(\frac{4}{5}\right)^{n-1}$

$U_9 = 450 \times \left(\frac{4}{5}\right)^{8}$

$U_9 = 75.497$

$\therefore U_9 \sim 75.5$

(3)

(b) $S_n = \frac{a(1-r^n)}{1-r}$

(2)

$S_{15} = \frac{450(1-0.8^{15})}{1-0.8} = 2170.835 \sim 2171$ awe hours..

(c) $S_\infty = \frac{a}{1-r}$

(2)

$S_\infty = \frac{450}{1-0.8}$

$= \frac{450}{0.2}$

$S_\infty = 2250$ hours in total.