Outcome 1 U2 Prac2

1

Q1.

$$f(x) = \tan^{-1}(x^3) \Rightarrow f'(x) = \frac{1}{1+(x^3)^2} \cdot 3x^2$$

$$= \frac{3x^2}{1+x^6}$$

2

Q2.

$$x^2 + y^3 = 1$$

$$2x + 3y^2 \frac{dy}{dx} = 0$$

$$3y^2 \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{3y^2}$$

3

Q 3.

$$x = t^2 + t \quad \& \quad y = t^4$$

$$\frac{dx}{dt} = 2t + 1 \qquad \frac{dy}{dt} = 4t^3$$

2

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t^3}{2t+1}$$

Threshold

$\left(\frac{5}{7}\right)$

Outcome 2 U2 Prac 2

Q4. $\dfrac{A}{x} + \dfrac{B}{(x+2)} = \dfrac{3x+8}{x(x+2)}$

$\therefore \quad A(x+2) + Bx = 3x+8$

Let $x=0$: $\quad 2A = 8$, $\quad A = 4$

Let $x=-2$: $\quad -2B = -6+8$, $\quad -2B = 2$, $\quad \therefore B = -1$

$$\therefore \int \frac{3x+8}{x(x+2)}\,dx = \int \left(\frac{4}{x} - \frac{1}{(x+2)} \right) dx$$

$$= 4\ln|x| - \ln|x+2| + C$$

$$= \ln\left|\frac{x^4}{x+2}\right| + C$$

3

Q5. $\displaystyle\int_0^{\pi} x\sin x\,dx = \left[-x\cos x\right]_0^{\pi} - \int_0^{\pi} -\cos x\,dx$

$u = x \quad v = -\cos x$

$u' = 1 \quad v' = \sin x$

$\int uv' = uv - \int u'v$

$$= \left[-x\cos x\right]_0^{\pi} + \int_0^{\pi} \cos x\,dx$$

$$= \left[-x\cos x + \sin x\right]_0^{\pi}$$

$$= \left[\left(-(\pi)\cos(\pi) + \sin(\pi)\right) - (0 + \sin 0)\right]$$

$$= (-\pi \times -1 + 0) - 0$$

$$= \pi$$

4

* Remember to differentiate the 'easier' function that can reduce down (Not 'loop' like sin/cos)

Outcome 2 U2 Prac 2

3

Q6

$$\frac{dy}{dx} = x^2 y$$

$$\int \frac{dy}{y} = \int x^2 \, dx$$

$$\ln|y| = \frac{x^3}{3} + c$$

$$y = e^{x^3/3 + c}$$

3

$$y = e^{x^3/3} \cdot e^c$$

$$\therefore y = Ae^{x^3/3}, \text{ where } A = e^c$$

Threshold

(7/10)

Outcome 3 U2 Prac 2

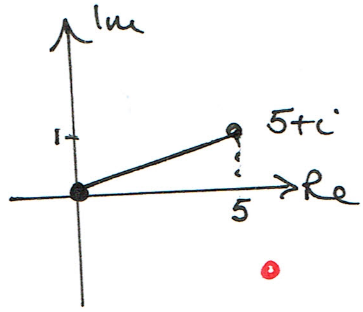
Q7. $z = 1 - i \; ; \; w = 2 + 3i \; ; \; u = \sqrt{3} + i$

a) $zw = (1-i)(2+3i)$

$= 2 + 3i - 2i - 3i^2$

$= 2 + i + 3$

$= 5 + i$



2

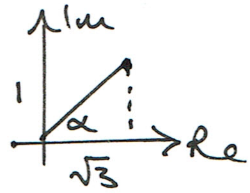
b) $u = \sqrt{3} + i$

$|u| = \sqrt{(\sqrt{3})^2 + 1^2}$

$= \sqrt{3+1}$

$= \sqrt{4}$

$= 2$



1st Quad $\alpha = \tan^{-1}\left|\frac{1}{\sqrt{3}}\right|$

$\arg(\alpha) = \frac{\pi}{6}$

3

$\therefore \quad u = r(\cos\theta + i\sin\theta)$

$u = 2\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$

Threshold
$(3/5)$

Outcome 4 U2 Prac 2

Q8. $3, 14, 25, \ldots \Rightarrow a = \underline{\underline{3}}$ & $d = 14 - 3 = \underline{\underline{11}}$

a) $\boxed{U_n = a + (n-1)d}$

$U_{50} = 3 + (50-1) \times 11$

$= 3 + 49 \times 11$

$\therefore U_{50} = 542$

b) $\boxed{S_n = \frac{n}{2}\left[2a + (n-1)d\right]}$

$S_n = \frac{n}{2}\left[2 \times 3 + (n-1) \times 11\right]$

$= \frac{n}{2}\left[6 + 11n - 11\right]$

$S_n = \frac{n}{2}\left[11n - 5\right]$

OR $S_n = \frac{11n^2 - 5n}{2}$

4

Q9. $5, 10, 20, \ldots \quad a = \underline{\underline{5}}$ & $r = \frac{10}{5} = \frac{20}{10} = \underline{\underline{2}}$

$\boxed{U_n = ar^{n-1}}$

$U_{15} = 5 \times 2^{14}$

$\therefore U_{15} = 81920$

b) $\boxed{S_n = \frac{a(1-r^n)}{1-r}}$

$S_{20} = \frac{5(1-2^{20})}{1-2}$

$= \frac{5(1-2^{20})}{-1}$

$\therefore S_{20} = 5242875$

4

Outcome 5 U2 Prac 2

6

Q10. $\boxed{\frac{a}{b} < 1 \Rightarrow a < b}$

Let $a = -1$ & $b = -3$

• Suitable Values

Then $\frac{a}{b} = \frac{-1}{-3} = \frac{1}{3} < 1$

3

$\boxed{\text{if } a = -1 \text{ \& } b = -3}$

• Disprove

then $-3 < -1$

$b < a$

• Comment

But initially stated $a < b$, so disproved and conjecture is false / not true.

Q11. $\boxed{\text{'}n^2 \text{ is odd, then } n \text{ is odd'}}$

4

Assume untrue, such that if 'n^2 is odd, then n is EVEN.'

•

Let $n = 2k \Rightarrow n^2 = (2k)^2 = 4k^2$

•

So $n^2 = 2(2k^2) \Rightarrow \boxed{\text{Even}}$

•

However assumed if n^2 is odd, n is EVEN.

But n^2 is EVEN. CONTRADICTION!!!

•

Thus by Proof of Contradiction initial conjecture true and if n^2 is odd, n must be odd.

Th $\left(\frac{5}{7}\right)$