Outcome 1 U2 Prac 1

Q1.

Recall, $f(x) = \sin^{-1}(x)$

then $f'(x) = \dfrac{1}{\sqrt{1-x^2}}$

$f(x) = \sin^{-1}(x^4)$ (2)

$\Rightarrow f'(x) = \dfrac{1}{\sqrt{1-(x^4)^2}} \cdot 4x^3$

$= \dfrac{4x^3}{\sqrt{1-x^8}}$ (should finish together)

Q2.

$x^2 + 5y^2 = 20$

$2x + 10y \cdot \dfrac{dy}{dx} = 0$

$10y \cdot \dfrac{dy}{dx} = -2x$

$\therefore \; dy/dx = \dfrac{-2x}{10y} \Rightarrow \boxed{\dfrac{dy}{dx} = \dfrac{-x}{5y}}$

(3)

Q 3.

$x = t^3 - t$ | $y = 2t^2$

$\dfrac{dx}{dt} = 3t^2 - 1$ | $\dfrac{dy}{dt} = 4t$

(2)

$\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{4t}{3t^2-1}$

Threshold $\left(\dfrac{5}{7}\right)$

Q4. $\int \frac{x-20}{x(x-5)}\,dx \Rightarrow \frac{A}{x} + \frac{B}{(x-5)} = \frac{x-20}{x(x-5)}$

$\therefore \quad A(x-5) + Bx = x-20$

Let $x=0$: $\quad -5A = -20$

$\Rightarrow A = 4$

Let $x=5$: $\quad 5B = 5-20$

$5B = -15$

$\Rightarrow B = -3$

③

$\therefore \quad \int \frac{x-20}{x(x-5)}\,dx = \int \left(\frac{4}{x} - \frac{3}{(x-5)} \right) dx$

$= 4\ln|x| - 3\ln|x-5| + C$ → This line is enough, but good prac. to present altogether ←

$= \ln|x^4| - \ln|(x-5)^3| + C$

$= \ln\left| \frac{x^4}{(x-5)^3} \right| + C$

Outcome 2 U2 Prac 1

$$\int uv' = uv - \int u'v$$

$u = 2x \qquad v = -\cos x$
$u' = 2 \qquad v' = \sin x$

Q5.
$$\int_0^{\pi/4} 2x \sin x \, dx = \left[-2x\cos x\right]_0^{\pi/4} - \int_0^{\pi/4} (-2\cos x)\,dx$$

$$= \left[-2x\cos x\right]_0^{\pi/4} + 2\int_0^{\pi/4} \cos x \, dx$$

$$= \left[-2x\cos x + 2\sin x\right]_0^{\pi/4}$$

$$= \left[-2\left(\tfrac{\pi}{4}\right)\cos\left(\tfrac{\pi}{4}\right) + 2\sin\left(\tfrac{\pi}{4}\right)\right] - \left[0 + 2\sin 0\right]$$

$$= \left(-\frac{\pi}{2} \times \frac{1}{\sqrt{2}} + 2 \times \frac{1}{\sqrt{2}}\right) - (0)$$

4

$$= \frac{2}{\sqrt{2}} - \frac{\pi}{2\sqrt{2}}$$

Alternative Rearrangements

$$= \frac{4-\pi}{2\sqrt{2}} \quad \text{or} \quad \frac{\sqrt{2}(4-\pi)}{2\sqrt{2}.\sqrt{2}} = \frac{\sqrt{2}(4-\pi)}{4}$$

Q6.
$$\frac{dy}{dx} = y\sin x$$

3

(If omit 'c' lose 2 marks instantly)

$$\int \frac{dy}{y} = \int \sin x \, dx$$

$$\ln|y| = -\cos x + c$$

$$y = e^{-\cos x + c} \quad (= e^{-\cos x}.e^{c})$$

$$y = Ae^{-\cos x}, \text{ where } A = e^{c}$$

Threshold $\left(\frac{7}{10}\right)$

Outcome 3 U2 Prac 1

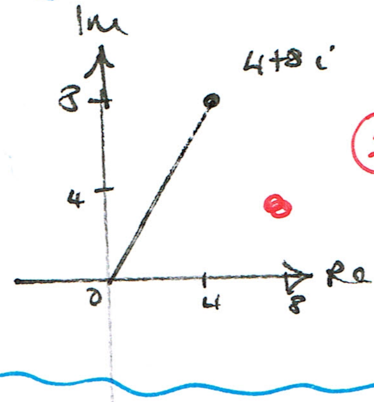
$z = 2+2i$, $w = 3+i$, $u = 3+\sqrt{3}i$

Q7. a) $zw = (2+2i)(3+i)$

$= 6 + 2i + 6i + 2i^2$

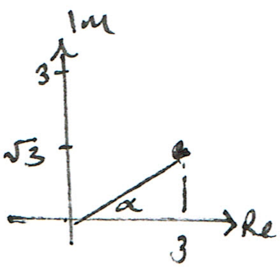
$= 6 + 8i - 2$

$= 4 + 8i$



2

b)



$u = 3+\sqrt{3}i$

$|u| = \sqrt{3^2+(\sqrt{3})^2}$

$= \sqrt{9+3}$

$= \sqrt{12}$

$\tan\alpha = |b/a|$

$\alpha = \tan^{-1}\left|\frac{\sqrt{3}}{3}\right|$

$= \tan^{-1}\left|\frac{1}{\sqrt{3}}\right|$

$\arg(u) = \frac{\pi}{6}$

If $u = r(\cos\theta + i\sin\theta)$

then $u = \sqrt{12}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

3

Threshold $\left(\frac{3}{5}\right)$

Q8. $7, 13, 19, \ldots$ $a = 7$; $d = 13 - 7 = 6$

a) $\boxed{U_n = a + (n-1)d}$

$U_{18} = 7 + (18-1) \times 6$

$= 7 + 17 \times 6$

$= 7 + 102$

$\therefore U_{18} = 109$

b) $\boxed{S_n = \frac{n}{2}\left[2a + (n-1)d\right]}$

$S_{18} = \frac{18}{2}\left[2 \times 7 + (18-1) \times 6\right]$

$= 9\left[14 + 102\right]$

$= 9 \times 116$

$= 1044$

4

Q9. $64, 16, 4 \ldots$ $a = 64$ $r = \frac{16}{64} = \frac{4}{16} = \frac{1}{4}$

$\boxed{U_n = ar^{n-1}}$

a) $U_9 = 64 \times \left(\frac{1}{4}\right)^8$

$= 64 \times \frac{1}{65536}$

$U_9 = \frac{1}{1024}$

$\boxed{S_n = \frac{a(1-r^n)}{1-r}}$

4

$S_n = \frac{64\left(1 - (1/4)^n\right)}{1 - 1/4}$

$= \frac{64\left(1 - \left(\frac{1}{4}\right)^n\right)}{\frac{3}{4}}$

$S_n = \frac{256\left(1 - (1/4)^n\right)}{3}$

Q10. $\boxed{\frac{a}{b} > 1 \Rightarrow a > b}$

Let $a = -3$; $b = -1$ • — Choose suitable values

Then $\frac{a}{b} = \frac{-3}{-1} = 3 > 1$ ✓ true

but $-3 < -1 \Rightarrow a < b$. • — Disprove

③

Initially stated $a > b$, thus since $a < b$ disproved and conjecture is NOT TRUE. • — Communicate

---

Q11. $\boxed{\text{Let } n \text{ be a natural number. '5n is odd, then n is odd'}}$

Assume statement is false.

Assume if $5n$ is odd, then n must be even. •

Let $\boxed{n = 2k}$, where n is even & k is a positive integer ($k \in Z^+$ or $k \in N$) •

Then $5n = 5(2k)$

$= 10k$

$= 2(5k)$ •

$= \text{Even}$

④

But we stated if n is even $5n$ would be odd. •

$\boxed{\text{CONTRADICTION!}}$ So if $5n$ is odd, n must be odd.

Thr. $\frac{5}{7}$