Prac NAB AH Unit 1 Threshold $\frac{3}{5}$ ①

come 1

. $(p+q)^4 = \binom{4}{0}(p)^4(q)^0 + \binom{4}{1}(p)^3(q)^1 + \binom{4}{2}(p)^2(q)^2$

$+ \binom{4}{3}(p)^1(q)^3 + \binom{4}{4}(p)^0(q)^4$

1
1
2 1
3 1
6 4 1

$= (1)p^4(1) + (4)p^3q + (6)p^2q^2 + (4)pq^3 + (1)q^4$

$= \underline{p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4}$

• coeffs • powers

$\boxed{2}$

. $\frac{3x-11}{(x-2)(x+3)} = \frac{A}{(x-2)} + \frac{B}{(x+3)}$ •

$A(x+3) + B(x-2) = 3x - 11$ •

er $x=2$: $A(2+3) + 0 = 3(2) - 11$

$5A = 6 - 11$

$5A = -5$

$\underline{A = -1}$

er $x=-3$: $0 + B(-3-2) = 3(-3) - 11$

$-5B = -9 - 11$

$-5B = -20$

$\underline{B = 4}$

$\boxed{3}$

$\therefore \frac{3x-11}{(x-2)(x+3)} = \frac{4}{(x+3)} - \frac{1}{(x-2)}$ •

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Outcome 2 (4/6)

Outcome 2 (4/6) ②

3(a) $f(x) = 2x^4 \ln x$

$u = 2x^4 \quad v = \ln x$
$u' = 8x^3 \quad v' = 1/x$

$f'(x) = u'v + uv'$

$= 8x^3 \ln x + 2x^4 \times \frac{1}{x}$

$= 8x^3 \ln x + 2x^3$ $\boxed{2}$

$= 2x^3(4\ln x + 1)$

(b) $f(x) = \frac{4x-1}{x+3}$

$u = 4x-1 \quad v = x+3$
$u' = 4 \quad v' = 1$

$f'(x) = \frac{u'v - uv'}{v^2}$

$f'(x) = \frac{4(x+3) - 1(4x-1)}{(x+3)^2}$

$= \frac{4x+12 - 4x+1}{(x+3)^2}$

$= \frac{13}{(x+3)^2}$

$\boxed{2}$

3(c) $f(x) = e^{(1-\cos x)}$

$f'(x) = e^{(1-\cos x)} \cdot \sin x$

$= \sin x . e^{(1-\cos x)}$ $\boxed{2}$

Outcome 3

$\frac{5}{7}$

4 (a) $\int \frac{3x^2}{x^3+2}\,dx$

Let $u = x^3+2$

$\frac{du}{dx} = 3x^2$

$\frac{du}{3x^2} = dx$

$= \int \frac{3x^2}{u} \cdot \frac{du}{3x^2}$

$= \int \frac{du}{u}$

$= \ln|u| + C$

$= \ln|x^3+2| + C$

$\boxed{2}$

(b) $\int e^{4x}\,dx = \frac{e^{4x}}{4} + C$

$\boxed{2}$

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5 $\int \cos^2 x \sin x\,dx$

Let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$\frac{du}{-\sin x} = dx$

$= \int (\cos x)^2 \sin x\,dx$

$= \int u^2 . \sin x \left(\frac{du}{-\sin x}\right)$

$= \int -u^2\,du$

$= \frac{-u^3}{3} + C$

$\boxed{3}$

$= -\frac{(\cos x)^3}{3} + C$ OR $-\frac{1}{3}\cos^3 x + C$

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Outcome 4

(6/9)

6(a) Vertical Asymptote (when undefined)

$$x + 3 = 0$$

$\therefore$ At $x = -3$

[1]

(b) Oblique / Non-Horizontal (when $x \to \pm\infty$)

$$y = \frac{x^2+4x+7}{x+3}$$

$$\begin{array}{r|l} & x \quad + 1 \\ \hline x+3 & x^2 + 4x + 7 \\ & x^2 + 3x \\ \hline & \qquad\quad x + 7 \\ & \qquad\quad x + 3 \\ \hline & \qquad\qquad\ 4 \end{array}$$

$$y = x + 1 + \frac{4}{x+3}$$

$\therefore$ Asymptote at $y = x + 1$

[2]

(c) Cuts Axes

$x = 0$: $y = \frac{(0)^2 + 4(0) + 7}{(0) + 3} = \frac{7}{3} \Rightarrow (0, 7/3)$

$y = 0$: $\frac{x^2+4x+7}{x+3} = 0$

$$x^2 + 4x + 7 = 0$$

$b^2 - 4ac$
$= (4)^2 - 4 \times 1 \times 7$
$= 16 - 28$
$= -12$

As $b^2 - 4ac < 0 \Rightarrow$ Doesn't cut x-axis at all.

6 (C)

mmetry not required in 'NAB' question. *

$$\frac{(-x)^2+4(-x)+7}{(-x)+3} = \frac{x^2-4x+7}{-x+3} = -\left(\frac{x^2-4x+7}{x-3}\right)$$

$\boxed{\text{As } f(-x) \neq \pm f(x) \Rightarrow \text{Not Odd/Even, so No Symmetry}}$

x/Min
es

$$y = \frac{x^2+4x+7}{x+3} = x+1+\frac{4}{(x+3)} = x+1+4(x+3)^{-1}$$

$$y = x+1+4(x+3)^{-1}$$

$$\frac{dy}{dx} = 1-4(x+3)^{-2}$$

$$= 1-\frac{4}{(x+3)^2}$$

at Pts
r $\frac{dy}{dx}=0$ $\Longrightarrow$

$$1-\frac{4}{(x+3)^2} = 0$$

$$1 = \frac{4}{(x+3)^2}$$

$$(x+3)^2 = 4$$

$$(x+3) = \pm 2$$

$$x = -3 \pm 2$$

$x = -3-2$
$x = \underline{\underline{-5}}$

$x = -3+2$
$\underline{\underline{x = -1}}$

ords

At $x=-5$: $y = (-5)+1+\frac{4}{(-5+3)} = -4+\frac{4}{(-2)} = \underline{\underline{-6}} \Rightarrow (-5,-6)$

At $x=-1$: $y = (-1)+1+\frac{4}{(-1+3)} = \frac{4}{(2)} = \underline{\underline{2}} \Rightarrow (-1,2)$

$\boxed{\text{So 2 Stat Pts Exist at } (-5,-6) \text{ \& } (-1,2)}$

6(c)

$$y = x + 1 + 4(x+3)^{-1}$$

$$\frac{dy}{dx} = 1 - 4(x+3)^{-2}$$

$$\frac{d^2y}{dx^2} = 8(x+3)^{-3} = \frac{8}{(x+3)^3}$$

Can use a NATURE TABLE or Use $\frac{d^2y}{dx^2}$

NATURE

$(-1, 2)$; $x = -1$:

$$\frac{d^2y}{dx^2} = \frac{8}{(-1+3)^3} = \frac{8}{(2)^3} = 1 > 0 \Rightarrow \cup$$

MIN TPt $(-1, 2)$

$(-5, -6)$; $x = -5$:

$$\frac{d^2y}{dx^2} = \frac{8}{(-5+3)^3} = \frac{8}{(-2)^3} = \frac{8}{-8} = -1 < 0 \Rightarrow \cap$$

MAX TPT $(-5, -6)$

Themes of Asymptotes

$$y = x + 1 + \frac{4}{(x+3)}$$

Vertical

$x \to -3^{+}$ $\quad y \to +\infty$

$x \to -3^{-}$ $\quad y \to -\infty$

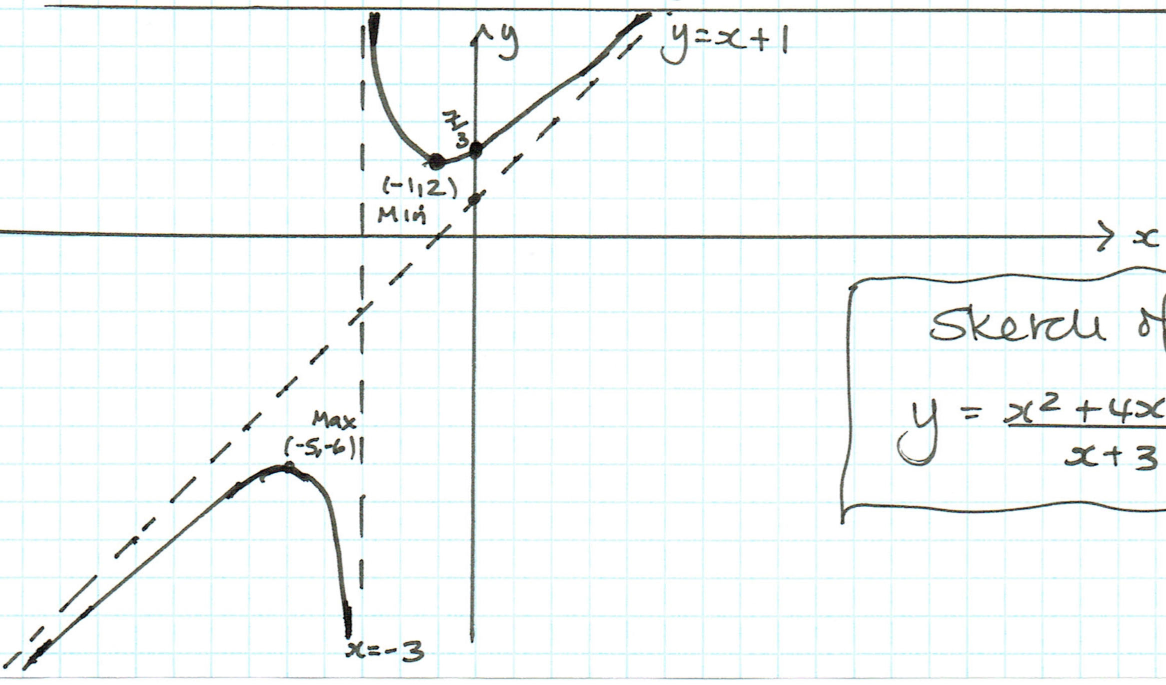
{ think -2.9 & -3.1

Non-Horizontal Oblique

$x \to +\infty$ $\quad y \to (x+1)^{+}$

$x \to -\infty$ $\quad y \to (x+1)^{-}$

6



Sketch of

$$y = \frac{x^2 + 4x + 7}{x + 3}$$

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Outcome 5

35

7.

$$\begin{aligned} x - y \quad &= 5 \\ 3y + z &= -7 \\ x - 2y - z &= 8 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & 5 \\ 0 & 3 & 1 & -7 \\ 1 & -2 & -1 & 8 \end{array}\right) \begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} \xrightarrow{r_3 - r_1} \left(\begin{array}{ccc|c} 1 & -1 & 0 & 5 \\ 0 & 3 & 1 & -7 \\ 0 & -1 & -1 & 3 \end{array}\right) \begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array}$$

$$3r_3 \quad \left(\begin{array}{ccc|c} 1 & -1 & 0 & 5 \\ 0 & 3 & 1 & -7 \\ 0 & -3 & -3 & 9 \end{array}\right) \begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} \rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 0 & 5 \\ 0 & 3 & 1 & -7 \\ 0 & 0 & -2 & 2 \end{array}\right) \begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array}$$

$\therefore \; -2z = 2$; $3y + z = -7$; $x - y = 5$

$\underline{\underline{z = -1}}$ $\quad 3y - 1 = -7 \quad x - (-2) = 5$

$3y = -6 \quad x + 2 = 5$

$\underline{\underline{y = -2}} \quad \underline{\underline{x = 3}}$

$\therefore$ Solution is Unique with 3 equations intersecting at $(3, -2, -1)$

$\boxed{5}$