AH Outcome 1 : Prac NAB U3

Q1. $\underline{a} = \underline{i} - \underline{j} - \underline{k}$

$\underline{b} = \underline{i} + 2\underline{j} + \underline{k}$

$$\underline{a} \times \underline{b} = \begin{pmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{pmatrix}$$

$$= \begin{vmatrix} -1 & -1 \\ 2 & 1 \end{vmatrix}\underline{i} - \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}\underline{j} + \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix}\underline{k}$$

$$= (-1-(-2))\underline{i} - (1-(-1))\underline{j} + (2-(-1))\underline{k}$$

$$= \underline{i} - 2\underline{j} + 3\underline{k}$$

3

Q2. $$\underline{d} = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ 5 \end{pmatrix}$$

$\boxed{r = a + \lambda d}$ $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -2 \\ 5 \end{pmatrix}$$

2

$\therefore$ $x = 1 - 2\lambda$; $y = 3 - 2\lambda$ & $z = -2 + 5\lambda$

Q3. $\boxed{r.n = a.n}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix}$$

$$3x - 2y - 2z = 6 + 2 - 6$$

2

$$\Rightarrow \quad 3x - 2y - 2z = 2$$

AH. Outcome 2 Prac NAB U3

Q4

a) $2K - L + 2M$

$$= 2\begin{bmatrix} 4 & a \\ 3 & -1 \end{bmatrix} - \begin{bmatrix} 6 & 2 \\ 0 & 1 \end{bmatrix} + 2\begin{bmatrix} 2 & b \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 2a \\ 6 & -2 \end{bmatrix} + \begin{bmatrix} -6 & -2 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 4 & 2b \\ 6 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 2a-2+2b \\ 12 & 5 \end{bmatrix}$$

2

b)

$$LM = \begin{bmatrix} 6 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & b \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 12+6 & 6b+8 \\ 0+3 & 0+4 \end{bmatrix}$$

$$\therefore LM = \begin{bmatrix} 18 & 6b+8 \\ 3 & 4 \end{bmatrix}$$

2

Q5.

$$\text{Det } G = 1\begin{vmatrix} 1 & -1 \\ 2 & -2 \end{vmatrix} - 4\begin{vmatrix} 2 & -1 \\ 0 & -2 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 1(-2-(-2)) - 4(-4-0) + 3(4-0)$$

$$= 1(0) + 16 + 12$$

$$= 28$$

2

Q6.

$$\text{Inverse } H^{-1} = \frac{1}{\text{Det } H}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{(4-(-2))}\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$$

2

$$H^{-1} = \tfrac{1}{6}\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} \text{ or } \begin{pmatrix} 1/6 & 1/3 \\ -1/6 & 2/3 \end{pmatrix}$$

Outcome 3 U3 Prac NAB

Q7. $f(x) = e^{4x}$ | $f(0) = e^0 = 1$

$f'(x) = 4e^{4x}$ | $f'(0) = 4$

$f''(x) = 16e^{4x}$ | $f''(0) = 16$

$f'''(x) = 64e^{4x}$ | $f'''(0) = 64$

$$\boxed{f(x) \simeq f(0) + f'(0)\frac{x}{1!} + f''(0)\frac{x^2}{2!} + \dots + f^n(0)\frac{x^n}{n!}}$$

3

$$\therefore f(x) = e^{4x} \simeq 1 + \frac{4x}{1!} + \frac{16x^2}{2!} + \frac{64x^3}{3!} + \dots$$

$$\therefore e^{4x} = 1 + 4x + 8x^2$$ (the first 3 terms only)

Q8. $$\boxed{x_{n+1} = 1 + \tfrac{1}{2}\ln(1+x_n), \quad x_0 = 2}$$

$x_0 = 2$

$x_1 = 1.549306$

$x_2 = 1.4679106$

$x_3 = 1.4576859$

$x_4 = 1.448388$

⋮

$x_n = 1.4447542161$ $\Rightarrow$ $x = 1.45$ to 2dps

$x_{n+1} = 1.4447542$

3

AH Outcome 4 : Prac NAB U3.

Q9.
$$\frac{dy}{dx} + \frac{1}{x}y = x^6$$

$$I(x) = e^{\int P(x)\,dx}$$
$$= e^{\int 1/x \, dx}$$
$$= e^{\ln|x|}$$
$$I(x) = x$$

$$x\left(\frac{dy}{dx} + \frac{1}{x}y\right) = x(x^6)$$

$$\boxed{\frac{d}{dx}(I(x)y) = I(x)f(x)}$$

$$\frac{d}{dx}(xy) = x(x^6)$$

$$\int \frac{d}{dx}(xy)\,.dx = \int x^7\,dx$$

$$xy = \frac{x^8}{8} + c$$

$$y = \frac{x^7}{8} + \frac{c}{x}$$

$$\left(\text{or } y = \frac{x^7}{8} + Cx^{-1}\right)$$

5

Q10. $\sum_{r=1}^{n} 6r = 3n(n+1)$

Let $n=1$

$6\times1 = 3(1+1)$

$6\times1 = 3\times2$

$6=6$ ✓

true LHS = RHS
thus true for $n=1$

Assume true for $n=k$

$$\sum_{r=1}^{n=k} 6r = 3k(k+1)$$

Consider $n=k+1$

$$\sum_{r=1}^{n=k+1} 6r \left(= \sum_{r=1}^{n=k} 6r \boxed{+\,6(k+1)} \right) = 3k(k+1) \boxed{+\,6(k+1)}$$

$$= 3k(k+1) + 6(k+1)$$

$$= (k+1)(3k+6)$$

$$= (k+1)(3(k+2))$$

$$= 3(k+1)(k+2)$$

$$= 3(k+1)((k+1)+1)$$

If let $N=(k+1)$ $\Rightarrow$ $= 3N(N+1)$

as required.

Statement As true for $n=1$, assumed true for $n=k$ and by Proof of Mathematical Induction also true for $n=k+1$. True $\forall n \in N$ and $\sum_{r=1}^{n} 6r = 3n(n+1)$ $\forall n \in N$.

AH Outcome 5 : Prac NAB U3

Q11. $\gcd(1696, 1504)$

$$1696 = 1.1504 + 192$$
$$1504 = 7.192 + 160$$
$$192 = 1.160 + 32$$
$$160 = 5.32 + 0$$

3

$\Rightarrow \gcd(1696, 1504) = 32$