AH ODEs PPQ Sols

2014

$$\boxed{4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0}$$

Auxiliary Equation

$$4m^2 - 4m + 1 = 0$$

$$(2m-1)^2 = 0$$

$$\therefore \quad m = \tfrac{1}{2} \text{ (twice)}$$

Complementary Function

$$y_c = Ae^{1/2x} + Bxe^{1/2x}$$

$x=0,\ y=4$

$$4 = Ae^0 + 0 \Rightarrow A = 4$$

$\frac{dy}{dx} = 3\ ;\ x=0$

$$\frac{dy}{dx} = \tfrac{1}{2}Ae^{1/2x} + \left(Be^{1/2x} + \tfrac{1}{2}Bxe^{1/2x}\right)$$

$$3 = \tfrac{1}{2}Ae^0 + \left(Be^0 + 0\right)$$

$$3 = \tfrac{1}{2}A + B$$

$$3 = \tfrac{1}{2}(4) + B$$

$$\therefore \quad B = 1$$

Thus $y_c = Ae^{1/2x} + Bxe^{1/2x}$

Becomes $y = 4e^{1/2x} + xe^{1/2x}$

* Last Mark includes Subst to find B + final solution.

6

2013 $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 9y = 4e^{3x}$

Aux Eqn $m^2 - 6m + 9 = 0$

$(m-3)^2 = 0$

$\therefore \quad m = 3$ (twice)

$y_c = Ae^{3x} + Bxe^{3x}$

Particular integral is based on $y_p = ke^{3x}$ But as e^{3x} appears twice in y_c $\Rightarrow$ must set $y_p = kx^2e^{3x}$

Let $y_p = kx^2e^{3x}$

$u = kx^2 \quad v = e^{3x}$

$u' = 2kx \quad v' = 3e^{3x}$

$\dfrac{dy_p}{dx} = 2kxe^{3x} + 3kx^2e^{3x}$

$= ke^{3x}(2x + 3x^2)$

$u = ke^{3x} \quad v = 2x + 3x^2$

$u' = 3ke^{3x} \quad v' = 2 + 6x$

$\dfrac{d^2y_p}{dx^2} = 3ke^{3x}(2x + 3x^2) + ke^{3x}(2 + 6x)$

$= 6kxe^{3x} + 9kx^2e^{3x} + 2ke^{3x} + 6kxe^{3x}$

$= 9kx^2e^{3x} + 12kxe^{3x} + 2ke^{3x}$

(Continued overpage)

2013 (continued)

$$\boxed{\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 4e^{3x}}$$

$$\left(9kx^2e^{3x} + 12kxe^{3x} + 2ke^{3x}\right) - 6\left(2kxe^{3x} + 3kx^2e^{3x}\right) + 9\left(kx^2e^{3x}\right) = 4e^{3x}$$

$$\cancel{9kx^2e^{3x}} + \cancel{12kxe^{3x}} + 2ke^{3x} - \cancel{12kxe^{3x}} - \cancel{18kx^2e^{3x}} + \cancel{9kx^2e^{3x}} = 4e^{3x}$$

$$\therefore \quad 2ke^{3x} = 4e^{3x}$$

$$2k = 4$$

$$\therefore \quad \underline{k = 2}$$

$$\therefore \quad y_p = kx^2e^{3x} \Rightarrow \underline{y_p = 2x^2e^{3x}}$$

$$\therefore \quad y = y_c + y_p$$

$$\underline{y = Ae^{3x} + Bxe^{3x} + 2x^2e^{3x}}$$

$\underline{x=0;\, y=1}$: $\quad 1 = Ae^0 + 0 + 0 \Rightarrow \underline{A=1}$

$$\frac{dy}{dx} = 3Ae^{3x} + \left(Be^{3x} + 3Bxe^{3x}\right) + \left(4xe^{3x} + 6x^2e^{3x}\right)$$

$\underline{x=0;\, \frac{dy}{dx}=-1}$: $\quad -1 = 3Ae^0 + \left(Be^0 + 0\right) + (0)$

$$-1 = 3 + B$$

$$\Rightarrow \underline{B = -4}$$

find B

and states solution

$$\therefore \quad \underline{y = e^{3x} - 4xe^{3x} + 2x^2e^{3x}}$$

OR

$$\underline{y = (2x^2 - 4x + 1)e^{3x}}$$

11

2012

a) $\dfrac{1}{(x-1)(x+2)^2} = \dfrac{A}{(x-1)} + \dfrac{B}{(x+2)} + \dfrac{C}{(x+2)^2}$ o

$\therefore \quad A(x+2)^2 + B(x-1)(x+2) + C(x-1) = 1$

Let $x=1$: $A(1+2)^2 = 1$

$9A = 1 \Rightarrow \underline{\underline{A = 1/9}}$ o

Let $x=-2$: $C(-2-1) = 1 \Rightarrow \underline{\underline{C = -1/3}}$ o

Let $x=0$: $4A - 2B - C = 1$

$4/9 - 2B + 1/3 = 1$

$\times 9$ $\quad 4 - 18B + 3 = 9$

$-18B = 2$

$-9B = 1 \Rightarrow \underline{\underline{B = -1/9}}$ o

$\therefore \quad \dfrac{1}{(x-1)(x+2)^2} = \dfrac{1/9}{(x-1)} - \dfrac{1/9}{(x+2)} - \dfrac{1/3}{(x+2)^2}$

$= \underline{\underline{\dfrac{1}{9(x-1)} - \dfrac{1}{9(x+2)} - \dfrac{1}{3(x+2)^2}}}$

4

Part (b) overpage

2012

(b) $$(x-1)\frac{dy}{dx} - y = \frac{(x-1)}{(x+2)^2}$$

$$\frac{dy}{dx} - \frac{y}{(x-1)} = \frac{1}{(x+2)^2}$$

$$I(x) = e^{\int P(x)\,dx}$$
$$= e^{\int \frac{-1}{(x-1)}dx}$$
$$= e^{-\ln|x-1|}$$
$$= e^{\ln|x-1|^{-1}}$$
$$= (x-1)^{-1}$$
$$= \frac{1}{(x-1)}$$

$\times$ by $I(x)$: $$\frac{1}{(x-1)}\left(\frac{dy}{dx} - \frac{y}{(x-1)}\right) = \frac{1}{(x-1)}\left(\frac{1}{(x+1)^2}\right)$$

$$\boxed{\frac{d}{dx}(I(x)\,y) = I(x)f(x)}$$

$$\frac{d}{dx}\left(\frac{1}{(x-1)}\cdot y\right) = \frac{1}{(x-1)(x+2)^2}$$

(from (a))

$$\int \frac{d}{dx}\left(\frac{y}{(x-1)}\right)dx = \int\left(\frac{1}{9(x-1)} - \frac{1}{9(x+2)} - \frac{1}{3(x+2)^2}\right)dx$$

$$\frac{y}{(x-1)} = \frac{1}{9}\int\frac{dx}{(x-1)} - \frac{1}{9}\int\frac{dx}{(x+2)} - \frac{1}{3}\int(x+2)^{-2}\,dx$$

$$\frac{y}{(x-1)} = \frac{1}{9}\ln|x-1| - \frac{1}{9}\ln|x+2| - \frac{1}{3}\left(\frac{(x+2)^{-1}}{-1}\right) + c$$

$$\frac{y}{(x-1)} = \frac{1}{9}\ln\left|\frac{x-1}{x+2}\right| + \frac{1}{3(x+2)} + c$$

$$\therefore \quad y = \left(\frac{1}{9}\ln\left|\frac{x-1}{x+2}\right| + \frac{1}{3(x+2)} + c\right)(x-1)$$

7

2011

AH ODEs PPQs

$$\boxed{\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^x + 12}$$

Aux Eqn

$$m^2 - m - 2 = 0$$

$$(m-2)(m+1) = 0$$

$$m = 2 \ \& \ m = -1$$

$$\therefore \ y_c = Ae^{2x} + Be^{-x}$$

Let $y_p = ke^x + c$

$$\frac{dy_p}{dx} = ke^x$$

$$\frac{d^2y_p}{dx^2} = ke^x$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^x + 12$$

$$(ke^x) - (ke^x) - 2(ke^x + c) = e^x + 12$$

$$ke^x - ke^x - 2ke^x - 2c = e^x + 12$$

$$-2ke^x - 2c = e^x + 12$$

$$\therefore \ -2ke^x = e^x \quad \& \quad -2c = 12$$

$$-2k = 1 \qquad c = -6$$

$$k = -1/2$$

$$\therefore \ y_p = -\tfrac{1}{2}e^x - 6$$

Finding k & c and subst

7

$$\therefore \ y = y_c + y_p = Ae^{2x} + Be^{-x} - \tfrac{1}{2}e^x - 6$$

This is the general solution. Particular solution overpage ↓

2011 (continued)

$$y = Ae^{2x} + Be^{-x} - \frac{1}{2}e^{x} - 6$$

$x=0,\ y=-\frac{3}{2}$

$$-\frac{3}{2} = Ae^{0} + Be^{0} - \frac{1}{2}e^{0} - 6$$

$$-\frac{3}{2} = A + B - \frac{1}{2} - 6$$

$$A + B = -\frac{3}{2} + \frac{1}{2} + 6$$

$$\therefore \quad A + B = 5 \quad \text{—} ①$$

$x=0;\ \frac{dy}{dx} = \frac{1}{2}$

$$\frac{dy}{dx} = 2Ae^{2x} - Be^{-x} - \frac{1}{2}e^{x}$$

$$\frac{1}{2} = 2Ae^{0} - Be^{0} - \frac{1}{2}e^{0}$$

$$\frac{1}{2} = 2A - B - \frac{1}{2}$$

3

$$\therefore \quad 2A - B = 1 \quad \text{—} ②$$

$$A + B = 5 \quad ①$$
$$2A - B = 1 \quad ②$$

$① + ②$: $3A = 6$

$A = 2$

Subst $A = 2$ into ①

$$A + B = 5$$
$$2 + B = 5$$
$$\Rightarrow \quad B = 3$$

$$\therefore \quad y = Ae^{2x} + Be^{-x} - \frac{1}{2}e^{x} - 6$$

$y = 2e^{2x} + 3e^{-x} - \frac{1}{2}e^{x} - 6$ is particular solution.

$$\boxed{\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0}$$

Aux. Eqn: $m^2 + 4m + 5 = 0 \Rightarrow$ Quad Formula!

$$\boxed{m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$m = \frac{-4 \pm \sqrt{(4)^2 - 4 \times 1 \times 5}}{2 \times 1}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm 2i}{2}$$

$\therefore \quad m = -2 \pm i \Rightarrow p = -2 \ \& \ q = 1$ (4)

$\therefore \quad y_c = e^{-2x}(A\cos x + B\sin x)$

$x = 0;\ y = 3$: $\quad 3 = e^0(A\cos 0 + B\sin 0)$

$3 = 1(A + 0) \Rightarrow A = 3$

$x = \frac{\pi}{2};\ y = e^{-\pi}$: $\quad e^{-\pi} = e^{-2(\pi/2)}\left(A\cos\left(\frac{\pi}{2}\right) + B\sin\left(\frac{\pi}{2}\right)\right)$

$e^{-\pi} = e^{-\pi}(3 \times 0 + B \times 1)$

$1 = B$

Thus G.Sol $y = e^{-2x}(A\cos x + B\sin x)$

becomes

Particular Solution: $y = e^{-2x}(3\cos x + \sin x)$ (3)

AH ODEs PPQs

2009. $(x+1)\dfrac{dy}{dx} - 3y = (x+1)^4$

$$\frac{dy}{dx} - \frac{3y}{(x+1)} = (x+1)^3$$

$$I(x) = e^{\int P(x)dx} = e^{\int \frac{-3}{(x+1)}dx} = e^{-3\ln(x+1)} = e^{\ln(x+1)^{-3}} = (x+1)^{-3} = \frac{1}{(x+1)^3}$$

× by $I(x)$: $\dfrac{1}{(x+1)^3}\left(\dfrac{dy}{dx} - \dfrac{3y}{(x+1)}\right) = \dfrac{1}{(x+1)^3}\left((x+1)^3\right)$

Becomes $\boxed{\dfrac{d}{dx}\left(I(x)\,y\right) = I(x)f(x)}$

$$\therefore \quad \frac{d}{dx}\left(\frac{1}{(x+1)^3}\cdot y\right) = \left(\frac{1}{(x+1)^3} \times (x+1)^3\right)$$

$$\int \frac{d}{dx}\left(\frac{1}{(x+1)^3}\cdot y\right)dx = \int 1\,dx$$

$$\frac{1}{(x+1)^3}\cdot y = x + C$$

Gen. sol^n: $y = (x+1)^3(x+C)$

$x=1;\ y=16$: $16 = (1+1)^3(1+C)$

$$16 = 8(1+C)$$

$$2 = 1 + C$$

$$\Rightarrow C = 1$$

6

Particular Solution $y = (x+1)^3(x+1)$

So $y = (x+1)^4$

2009 (Continued)

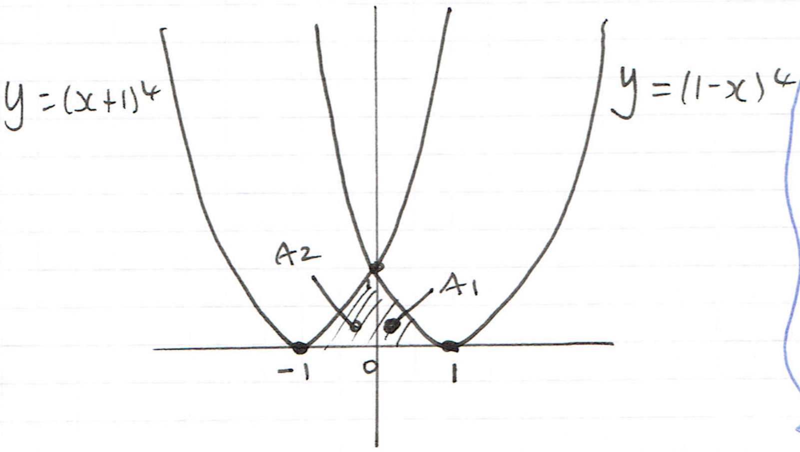
a) Found $y = (x+1)^4$

Intersect : $(x+1)^4 = (1-x)^4$
$x+1 = 1-x$
$2x = 0$
$x = 0$

b) Area Enclosed by $y=(x+1)^4$
& $y=(1-x)^4$ & x-axis

$y=(x+1)^4$ has a line of symmetry at $x=-1$
& cuts at $(-1,0)$ and $(0,1)$

$y=(1-x)^4$ has a line of symmetry at $x=1$
& cuts at $(1,0)$ & $(0,1)$



Symmetrical, so

find $\int_{-1}^{0}$ & double A_2

If use $\int_0^1$ care with chain rule!

$$A_2 = \int_{-1}^{0} (x+1)^4\,dx = \left[\frac{(x+1)^5}{5}\right]_{-1}^{0}$$

4

$$= \left(\frac{(0+1)^5}{5}\right) - \left(\frac{(-1+1)^5}{5}\right)$$

$$= \tfrac{1}{5} - 0$$

$$= \tfrac{1}{5}$$

$\therefore$ Total Area $= 2 \times \tfrac{1}{5} = \tfrac{2}{5}$ units^2

2008

$$\boxed{\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2}$$

Aux Eqn :

$$m^2 - 3m + 2 = 0$$

$$(m-2)(m-1) = 0$$

$$m = 2 \; ; \; m = 1$$

$$\therefore \quad y_c = Ae^{2x} + Be^{x}$$

Let $y_p = ax^2 + bx + c$

$$\frac{dy_p}{dx} = 2ax + b$$

$$\frac{d^2y_p}{dx^2} = 2a$$

$$\frac{d^2y}{dx^2} - \frac{3dy}{dx} + 2y = 2x^2$$

$$(2a) - 3(2ax+b) + 2(ax^2+bx+c) = 2x^2$$

$$2a - 6ax - 3b + 2ax^2 + 2bx + 2c = 2x^2$$

$$2ax^2 + (2b-6a)x + (2a-3b+2c) = 2x^2$$

$\therefore \quad 2ax^2 = 2x^2 \; ; \quad 2b - 6a = 0 \; ; \quad 2a - 3b + 2c = 0$

$\Rightarrow \; a = 1$

$2b - 6 = 0$

$2b = 6$

$b = 3$

$2 - 9 + 2c = 0$

$2c = 7$

$c = 7/2$

$$\therefore \quad y_p = x^2 + 3x + 7/2$$

$$y = y_c + y_p = Ae^{2x} + Be^{x} + x^2 + 3x + 7/2 \qquad 7$$

This is the General Solution.

(Particular Solution on next page)

$$\text{Given} \quad y = Ae^{2x} + Be^{x} + x^2 + 3x + \frac{7}{2}$$

$x = 0;\ y = \frac{1}{2}$

$$\frac{1}{2} = Ae^0 + Be^0 + 0 + 0 + \frac{7}{2}$$

$$\frac{1}{2} = A + B + \frac{7}{2}$$

$$\therefore \quad A + B = -3 \quad \text{—①}$$

$$\frac{dy}{dx} = 2Ae^{2x} + Be^{x} + 2x + 3$$

$x = 0;\ \frac{dy}{dx} = 1$:

$$1 = 2Ae^0 + Be^0 + 0 + 3$$

$$1 = 2A + B + 3$$

$$\therefore \quad 2A + B = -2 \quad \text{—②}$$

$$A + B = -3 \quad \text{—①}$$

$$2A + B = -2 \quad \text{—②}$$

② − ① $A = 1$

(3)

Subst $A = 1$ into ①

$$A + B = -3$$

$$1 + B = -3$$

$$\therefore \quad B = -4$$

Particular Solution:

$$y = e^{2x} - 4e^{x} + x^2 + 3x + \frac{7}{2}$$

2007

$$\boxed{\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = e^{2x}}$$

Aux Eqⁿ: $m^2 + 6m + 9 = 0$

$$(m+3)^2 = 0$$

$$\therefore \; m = -3 \text{ (twice)}$$

$$\therefore \; y_c = Ae^{-3x} + Bxe^{-3x}$$

As does not match f(x) no need for repeated roots to consider for y_p. (Just let $y_p = ke^{2x}$)

$$y_p = ke^{2x}$$

$$\frac{dy_p}{dx} = 2ke^{2x}$$

$$\frac{d^2y_p}{dx^2} = 4ke^{2x}$$

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = e^{2x}$$

$$(4ke^{2x}) + 6(2ke^{2x}) + 9(ke^{2x}) = e^{2x}$$

$$4ke^{2x} + 12ke^{2x} + 9ke^{2x} = e^{2x}$$

$$25ke^{2x} = e^{2x}$$

$$25k = 1$$

$$\therefore \; k = 1/25$$

$$\therefore \; y_p = \frac{1}{25}e^{2x}$$

General Solution: $y = Ae^{-3x} + Bxe^{-3x} + \frac{1}{25}e^{2x}$

6

(No conditions provided to find Particular Solution)

2006

$$\boxed{\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0}$$

Aux Eq^n: $m^2 + 2m + 2 = 0$ (Doesn't factorise! Use Quad Form)

$$\boxed{m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$\therefore \quad m = \frac{-2 \pm \sqrt{(2)^2 - 4\times1\times2}}{2\times1}$$

$$= \frac{-2 \pm \sqrt{-4}}{2}$$

$$\therefore \quad m = \frac{-2 \pm 2i}{2}$$

So $m = -1 \pm i$ $(p = -1;\ q = 1)$

$$y_c = e^{-x}(A\cos x + B\sin x)$$

$x = 0;\ y = 0$: $\quad 0 = e^0(A\cos 0 + B\sin 0)$

$$0 = 1(A + 0)$$

$$\therefore \underline{A = 0}$$

$\frac{dy}{dx} = 2;\ x = 0$: $\quad \frac{dy}{dx} = -e^{-x}(A\cos x + B\sin x) + e^{-x}(-A\sin x + B\cos x)$

$$2 = -e^0(A\cos 0 + B\sin 0) + e^0(-A\sin 0 + B\cos 0)$$

$$2 = -1(A + 0) + 1(0 + B)$$

$$2 = 0 + B \Rightarrow \underline{B = 2}$$

6

$$\therefore \quad y = e^{-x}(0 + 2\sin x)$$

$$\therefore \quad y = 2e^{-x}\sin x$$

2005 $\boxed{\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = 20\sin x}$

Aux Eqn: $m^2 - 3m + 2 = 0$

$(m-2)(m-1) = 0$

$\downarrow \quad \downarrow$

$m = 2$ & $m = 1$

$\therefore y_c = Ae^{2x} + Be^{x}$

$$\boxed{\begin{aligned} \text{Let } y_p &= p\sin x + q\cos x \\ \frac{dy_p}{dx} &= p\cos x - q\sin x \\ \frac{d^2y_p}{dx^2} &= -p\sin x - q\cos x \end{aligned}}$$

$\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = 20\sin x$

$(-p\sin x - q\cos x)$
$-3(p\cos x - q\sin x)$
$+2(p\sin x + q\cos x)$
$= 20\sin x$

$\downarrow$

$-p\sin x - q\cos x - 3p\cos x + 3q\sin x + 2p\sin x + 2q\cos x = 20\sin x$

$(-p + 3q + 2p)\sin x + (-q - 3p + 2q)\cos x = 20\sin x$

$(p + 3q)\sin x + (-3p + q)\cos x = 20\sin x$

$\therefore \quad p + 3q = 20 \quad$ — ①

$-3p + q = 0 \quad$ — ②

② ×3 $\quad -9p + 3q = 0$

① $\quad p + 3q = 20$

$-10p = -20$

$\therefore p = 2$

If $p = 2$ subst into ①

$p + 3q = 20$

$2 + 3q = 20$

$3q = 18$

$q = 6$

7

$y_p = 2\sin x + 6\cos x$

AH ODEs PPQs

2005 $\quad y = y_c + y_p$

$$\therefore \quad y = Ae^{2x} + Be^{x} + 2\sin x + 6\cos x$$

$x=0;\ y=0$

$$0 = Ae^{0} + Be^{0} + 2\sin 0 + 6\cos 0$$

$$0 = A + B + 0 + 6$$

$$\therefore \quad A + B = -6 \quad \text{—} (1)$$

$x=0;\ \dfrac{dy}{dx}=0:$

$$\frac{dy}{dx} = 2Ae^{2x} + Be^{x} + 2\cos x - 6\sin x$$

$$0 = 2Ae^{0} + Be^{0} + 2\cos 0 - 6\sin 0$$

$$0 = 2A + B + 2 - 0$$

$$\therefore \quad 2A + B = -2 \quad \text{—} (2)$$

$$A + B = -6 \quad \text{—} (1)$$

$$2A + B = -2 \quad \text{—} (2)$$

(2) − (1): $\quad A = 4$

3

Subst $A=4$ into (1)

$$4 + B = -6$$

$$\therefore \quad B = -10$$

Particular Solution:

$$y = 4e^{2x} - 10e^{x} + 2\sin x + 6\cos x$$

2004

$$\boxed{x\frac{dy}{dx} - 3y = x^4}$$

$$\frac{dy}{dx} - \frac{3y}{x} = x^3$$

$$I(x) = e^{\int P(x)dx}$$

$$= e^{\int -3/x\,dx}$$

$$= e^{-3\ln x}$$

$$= e^{\ln x^{-3}}$$

$$= x^{-3}$$

$$= 1/x^3$$

× by $I(x)$: $\frac{1}{x^3}\left(\frac{dy}{dx} - \frac{3y}{x}\right) = \frac{1}{x^3}(x^3)$

Produces $\frac{d}{dx}(I(x)y) = I(x)f(x)$

$$\frac{d}{dx}\left(\frac{1}{x^3}\cdot y\right) = \frac{1}{x^3}(x^3)$$

Integrate: $\int \frac{d}{dx}\left(\frac{1}{x^3}\cdot y\right)dx = \int 1\,dx$

$$\frac{1}{x^3}\cdot y = x + c$$

(5)

$$y = x^4 + cx^3$$

General Solution: $y = x^3(x+c)$

$y=2;\ x=1$: $2 = 1^3(1+c)$

$$2 = 1 + c$$

$$\Rightarrow c = 1$$

(2)

Particular Solution

$$y = x^3(x+1)$$

AH ODEs PPQs

2004 (18)

$$y\frac{dy}{dx} - 3x = x^4$$

$$\cancel{\frac{dy}{dx} - \frac{3x}{y}} = \frac{x^4}{y}$$

← doesn't help must be something else. Can you separate x & y?

$$y\frac{dy}{dx} - 3x = x^4$$

$$y\frac{dy}{dx} = x^4 + 3x$$

$$\int y\,dy = \int (x^4 + 3x)\,dx$$

$$\frac{y^2}{2} = \frac{x^5}{5} + \frac{3x^2}{2} + C$$

$$y^2 = 2\left(\frac{x^5}{5} + \frac{3x^2}{2} + C\right)$$

$$\therefore \quad y = \pm\sqrt{2\left(\frac{x^5}{5} + \frac{3x^2}{2} + C\right)}$$ is General Solution

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(4)

$y = 2\,;\ x = 1$: $\quad 2 = \pm\sqrt{2\left(\tfrac{1}{5} + \tfrac{3}{2} + C\right)}$

$$4 = 2\left(\tfrac{1}{5} + \tfrac{3}{2} + C\right)$$

$$2 = \left(\frac{2}{10} + \frac{15}{10} + C\right)$$

$$\therefore \quad C = 3/10$$

(This is fine not nec. to continue)

Particular Solution: $y = \pm\sqrt{2\left(\frac{x^5}{5} + \frac{3x^2}{2} + \frac{3}{10}\right)} = \pm\sqrt{2\left(\frac{2x^5 + 15x^2 + 3}{10}\right)}$

$$\therefore \quad y = \pm\sqrt{\frac{(2x^5 + 15x^2 + 3)}{5}}$$

2003

$$\boxed{\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^x}$$

Aux Eqn:

$$m^2 - 4m + 4 = 0$$
$$(m-2)^2 = 0$$
$$m = 2 \text{ (twice)}$$

$$\therefore y_c = Ae^{2x} + Bxe^{2x}$$

$f(x) \neq ke^{2x}$ so no need to worry about repeated roots for Particular Integral

Let $y_p = ke^x$

$$\frac{dy_p}{dx} = ke^x$$

$$\frac{d^2y_p}{dx^2} = ke^x$$

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^x$$

$$(ke^x) - 4(ke^x) + 4(ke^x) = e^x$$

$$\therefore ke^x = e^x$$

$$\Rightarrow k = 1$$

$$\therefore y_p = ke^x \text{ gives } y_p = e^x$$

$$y = (y_c + y_p) = Ae^{2x} + Bxe^{2x} + e^x$$

$x = 0;\ y = 2$:

$$2 = Ae^0 + 0 + e^0$$
$$2 = A + 1$$
$$\Rightarrow A = 1$$

10

$x = 0;\ \frac{dy}{dx} = 1$:

$$\frac{dy}{dx} = 2Ae^{2x} + (Be^{2x} + 2Bxe^{2x}) + e^x$$

$$1 = 2Ae^0 + Be^0 + 0 + e^0$$
$$1 = 2 + B + 1$$
$$\therefore B = -2 \quad \Rightarrow \quad y = e^{2x} - 2xe^{2x} + e^x$$

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$$\boxed{\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 4\cos x}$$

Aux Eqn: $m^2 + 2m + 5 = 0$ (Quad Formula!)

$$m = \frac{-2 \pm \sqrt{(2)^2 - 4\times1\times5}}{2\times1}$$

$$= \frac{-2 \pm \sqrt{-16}}{2}$$

$$\therefore\ m = \frac{-2 \pm 4i}{2}$$

$$m = -1 \pm 2i \quad (\Rightarrow p = -1 \ \&\ q = 2)$$

$$y_c = e^{-x}(A\cos 2x + B\sin 2x)$$

Let $y_p = p\sin x + q\cos x$

$dy_p/dx = p\cos x - q\sin x$

$d^2y_p/dx^2 = -p\sin x - q\cos x$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 4\cos x$$

$$(-p\sin x - q\cos x) + 2(p\cos x - q\sin x) + 5(p\sin x + q\cos x) = 4\cos x$$

$$(-p - 2q + 5p)\sin x + (-q + 2p + 5q)\cos x = 4\cos x$$

$$(4p - 2q)\sin x + (2p + 4q)\cos x = 4\cos x$$

$4p - 2q = 0 \quad -①$

$2p + 4q = 4 \quad -②$

① x 2: $8p - 4q = 0$

$2p + 4q = 4$

$10p = 4$

$p = 4/10 = 2/5$

Subst into ① $p = 2/5$

$4p - 2q = 0$

$4(2/5) - 2q = 0$

$8/5 = 2q$

$\therefore\ q = 4/5$

6

$$y_p = \frac{2}{5}\sin x + \frac{4}{5}\cos x$$

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$$y = y_c + y_P$$

$$\therefore \quad y = e^{-x}(A\cos 2x + B\sin 2x) + \frac{2}{5}\sin x + \frac{4}{5}\cos x$$

$y(0)=0$

$$0 = e^0(A\cos 0 + B\sin 0) + \frac{2}{5}\sin 0 + \frac{4}{5}\cos 0$$

$$0 = 1\,(A + 0) + 0 + 4/5$$

$$0 = A + 4/5$$

$$\Rightarrow \quad A = -\frac{4}{5}$$

$y'(0)=1$

$$\frac{dy}{dx} = e^{-x}(-2A\sin 2x + 2B\cos 2x) + \frac{2}{5}\cos x - \frac{4}{5}\sin x$$

$\Rightarrow x=0$

$y'(0)=1$

$$1 = e^0(-2A\sin 0 + 2B\cos 0) + \frac{2}{5}\cos 0 - \frac{4}{5}\sin 0$$

$$1 = 1\,(0 + 2B) + \frac{2}{5} - 0$$

$$1 = 2B + 2/5$$

$$2B = 3/5$$

$$\Rightarrow \quad B = 3/10$$

Particular Solution

4

$$y = e^{-x}\left(-4/5\cos 2x + \frac{3}{10}\sin 2x\right) + \frac{2}{5}\sin x + \frac{4}{5}\cos x$$

OR $$y = \frac{1}{10}\left(e^{-x}(-8\cos 2x + 3\sin 2x) + 4\sin x + 8\cos x\right)$$

(Not nec to do this as doesn't really make it any 'nicer')