AH: Implicit/Parametric Hwk ①

Q1. $xy + y^2 = 2$

(a) $\left(y + x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$

(xy)
$u = x \quad v = y$
$u' = 1 \quad v' = \frac{dy}{dx}$
$u'v + uv' = y + x\frac{dy}{dx}$

$(x+2y)\frac{dy}{dx} = -y$

$\frac{dy}{dx} = \frac{-y}{x+2y}$ ③

(b) Gradient, $m = \frac{dy}{dx}$ at $(1,1)$

$\therefore m = \frac{dy}{dx} = \frac{-y}{x+2y} = \frac{-1}{1+2} = \frac{-1}{3}$

$\boxed{y - b = m(x-a)}$ $m = \frac{-1}{3}$ at $(1,1)$

$(y-1) = \frac{-1}{3}(x-1)$ ②

$3y - 3 = -x + 1$

$\therefore x + 3y - 4 = 0$

AH Implicit / Parametric Hwk ②

As $A(-1,5)$ lies on curve subst $x=-1$; $y=5$ to find value of t that satisfies x & y

$x = t^2+t-1$
$-1 = t^2+t-1$
$t^2+t=0$
$t(t+1)=0$
$t=0$ & $t=-1$ •

$y = 2t^2-t+2$
$5 = 2t^2-t+2$
$2t^2-t-3=0$
$(2t-3)(t+1)=0$
$t=3/2$ & $t=-1$ •

∴ $t=-1$ satisfies both x & y at $A(-1,5)$ •

$\frac{dx}{dt} = 2t+1$; $\frac{dy}{dt} = 4t-1$ •

Gradient, $m = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t-1}{2t+1} = \frac{-4-}{-2+}$

∴ $m = \frac{-5}{-1} = 5$ •

$m=5$
$A(-1,5)$

$(y-b) = m(x-a)$
$(y-5) = 5(x+1)$
$y-5 = 5x+5$
∴ $y = 5x+10$ •

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Q3.

$$y^3 + 3xy = 3x^2 - 5$$

$$3y^2 \frac{dy}{dx} + 3y + 3x\frac{dy}{dx} = 6x$$

$$(3y^2 + 3x)\frac{dy}{dx} = 6x - 3y$$

$$\frac{dy}{dx} = \frac{6x-3y}{3y^2+3x} = \frac{2x-y}{y^2+x}$$

$3xy$

$u = 3x \quad v = y$

$u' = 3 \quad v' = dy/dx$

$u'v + uv'$

$3y + 3x\frac{dy}{dx}$

At $(2,1)$ $\quad m = \frac{dy}{dx} = \frac{2(2)-(1)}{(1)^2+(2)} = \frac{4-1}{1+2} = \frac{3}{3} = \underline{\underline{1}}$

$m = 1$ & $A(2,1)$ Equation of Tangent :-

$$y - b = m(x - a)$$

$$(y - 1) = 1(x - 2)$$

$$y - 1 = x - 2$$

$$y = x - 1 \quad \text{or} \quad x - y - 1 = 0$$

Old.

Q3. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-xy^2}{-x^2y} = \underline{\underline{\frac{y}{x}}}$ 1

Only asked to find dy/dx here

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Q4.

$x = 5\cos\theta \qquad y = 5\sin\theta$

$\frac{dx}{d\theta} = -5\sin\theta \qquad \frac{dy}{d\theta} = 5\cos\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{5\cos\theta}{-5\sin\theta} = \frac{-1}{\tan\theta} = -\cot\theta$$

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$$m = \frac{dy}{dx} = \frac{-1}{\tan\theta} = \frac{-1}{\tan\left(\frac{\pi}{4}\right)} = \frac{-1}{1} = -1$$

$$x = 5\cos\left(\frac{\pi}{4}\right) = 5 \times \frac{1}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

$$y = 5\sin\theta = 5\sin\left(\frac{\pi}{4}\right) = 5 \times \frac{1}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

$\therefore \quad \left(5/\sqrt{2},\ 5/\sqrt{2}\right)$ & $m = -1$

$$\left(y - \frac{5}{\sqrt{2}}\right) = -1\left(x - \frac{5}{\sqrt{2}}\right)$$

$$y - \frac{5}{\sqrt{2}} = -x + \frac{5}{\sqrt{2}}$$

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$$x + y - \frac{10}{\sqrt{2}} = 0$$

$$x + y - \frac{5\sqrt{2}\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}} = 0 \implies x + y = 5\sqrt{2}$$

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Q5.

$$2y^2 - 2xy - 4y + x^2 = 0$$

$$4y\frac{dy}{dx} - 2y - 2x\frac{dy}{dx} - 4\frac{dy}{dx} + 2x = 0$$

$$(4y - 2x - 4)\frac{dy}{dx} = 2y - 2x$$

$$\frac{dy}{dx} = \frac{2y - 2x}{4y - 2x - 4} = \frac{y - x}{2y - x - 2}$$

Horizontal tangent $\Rightarrow m = 0$

$$\therefore \frac{dy}{dx} = \frac{y - x}{2y - x - 2} = 0$$

$$\therefore y - x = 0$$

$$y = x$$

Use back in original eqn to obtain x

$$2y^2 - 2xy - 4y + x^2 = 0$$

$$2(x)^2 - 2x(x) - 4(x) + x^2 = 0$$

$$\cancel{2x^2} - \cancel{2x^2} - 4x + x^2 = 0$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

$$x = 0 \qquad x = 4$$

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Q6.

$$xy - x = 4$$

$$y + x\frac{dy}{dx} - 1 = 0$$

$$x\frac{dy}{dx} = 1 - y$$

$$\frac{dy}{dx} = \frac{1-y}{x}$$

2

xy: $u = x \quad v = y$

$u' = 1 \quad v' = \frac{dy}{dx}$

$u'v + uv'$

$= \left(y + x\frac{dy}{dx}\right)$

$u = 1 - y \quad v = x$

$u' = -\frac{dy}{dx} \quad v' = 1$

$$\frac{d^2y}{dx^2} = \frac{u'v - uv'}{v^2}$$

$$= \frac{-x\frac{dy}{dx} - 1(1-y)}{x^2}$$

$$= \frac{-\cancel{x}\left(\frac{1-y}{\cancel{x}}\right) - 1 + y}{x^2}$$

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$$= \frac{-1 + y - 1 + y}{x^2}$$

$$= \frac{2y - 2}{x^2}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{2(y-1)}{x^2}$$

AH Implicit/Parametric Hwk (7)

Q7. $\frac{x^2}{y} + x = y - 5$

$$u = x^2 \quad v = y$$
$$u' = 2x \quad v' = \frac{dy}{dx}$$

$$\frac{u'v - uv'}{v^2}$$

$$\frac{2xy - x^2\frac{dy}{dx}}{y^2} + 1 = \frac{dy}{dx}$$

$$2xy - x^2\frac{dy}{dx} + y^2 = y^2\frac{dy}{dx}$$

$$2xy + y^2 = (y^2 + x^2)\frac{dy}{dx}$$

$$(x^2 + y^2)\frac{dy}{dx} = y(2x + y)$$

$$\frac{dy}{dx} = \frac{y(2x+y)}{(x^2+y^2)}$$

At $(3,-1)$ $\quad m = \frac{-1(2\times3+(-1))}{((3)^2+(-1)^2)} = \frac{-1(6-1)}{(9+1)}$

$$m = \frac{-5}{10} = -\frac{1}{2}$$

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AH Implicit/Parametric Hwk (8)

Q8. $x = \cos 2t \qquad y = \sin 2t$

(a) $\frac{dx}{dt} = -2\sin 2t \qquad \frac{dy}{dt} = 2\cos 2t$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos 2t}{-2\sin 2t} = -\frac{1}{\tan 2t}$ or $-\cot 2t$

At $t = \frac{\pi}{8}$ $\quad m = \frac{dy}{dx} = \frac{-1}{\tan\left(2\left(\frac{\pi}{8}\right)\right)} = \frac{-1}{\tan\left(\frac{\pi}{4}\right)} = -1$

$x = \cos 2t$
$= \cos 2(\pi/8)$
$= \cos\left(\frac{\pi}{4}\right)$
$= \frac{1}{\sqrt{2}}$

$y = \sin 2t$
$= \sin 2(\pi/8)$
$= \sin(\pi/4)$
$= \frac{1}{\sqrt{2}}$

$(1/\sqrt{2}, 1/\sqrt{2})$ & $m = -1$ (5)

$(y - b) = m(x - a)$

$(y - 1/\sqrt{2}) = -1(x - 1/\sqrt{2})$

$y - 1/\sqrt{2} = -x + 1/\sqrt{2}$

$x + y = \frac{2}{\sqrt{2}} = \frac{\sqrt{2}\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$

$\therefore x + y = \sqrt{2}$

A4 Implicit / Parametric Hwk

Q8(b) from previous page $\frac{dy}{dx} = \frac{-\cos(2t)}{\sin(2t)}$ & $\frac{dx}{dt} = -2\sin(2t)$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\left(\frac{u'v-uv'}{v^2}\right)}{\frac{dx}{dt}} = \frac{\frac{2\sin^2(2t)+2\cos^2(2t)}{(\sin(2t))^2}}{(-2\sin(2t))}$$

$u = -\cos(2t)$ $\quad v = \sin(2t)$
$u' = 2\sin(2t)$ $\quad v' = 2\cos(2t)$

$$= \frac{\frac{2\left(\sin^2(2t)+\cos^2(2t)\right)}{\sin^2(2t)}}{-2\sin(2t)}$$

$$= \frac{2\ (1)}{-2\sin^3(2t)}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{-1}{\sin^3(2t)}$$

$$\sin(2t)\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = \sin(2t)\left(\frac{-1}{\sin^3(2t)}\right) + \left(\frac{-\cos(2t)}{\sin(2t)}\right)^2$$

$$= \frac{-1}{\sin^2(2t)} + \frac{\cos^2(2t)}{\sin^2(2t)}$$

$\left(\sin^2 + \cos^2 = 1 \quad \therefore \sin^2 = 1 - \cos^2\right)$

$$= \frac{(\cos^2(2t) - 1)}{\sin^2(2t)}$$

$$= \frac{\sin^2(2t)}{\sin^2(2t)} = 1 \quad \therefore k = 1$$

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(10)

Q9. $x = 2\sec\theta$ $\qquad y = 3\sin\theta$

$= \dfrac{2}{\cos\theta}$

$= 2(\cos\theta)^{-1}$

$dx/d\theta = -2(\cos\theta)^{-2}.\ -\sin\theta \qquad \dfrac{dy}{d\theta} = 3\cos\theta$

$= \dfrac{2\sin\theta}{\cos^2\theta}$

$\therefore \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{3\cos\theta}{\left(\dfrac{2\sin\theta}{\cos^2\theta}\right)} = \dfrac{3\cos^3\theta}{2\sin\theta}$

$\dfrac{dy}{dx} = \dfrac{3\cos^3\theta}{2\sin\theta} = \dfrac{3}{2}\cos^2\theta\cot\theta$

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Q10.

$$xy^2 + 3x^2y = 4$$

$$\left(y^2 + 2xy\frac{dy}{dx}\right) + \left(6xy + 3x^2\frac{dy}{dx}\right) = 0$$

$$(2xy + 3x^2)\frac{dy}{dx} = -6xy - y^2$$

$$\frac{dy}{dx} = \frac{-y(6x+y)}{x(3x+2y)}$$

xy^2

$u = x \quad v = y^2$

$u' = 1 \quad v' = 2y\frac{dy}{dx}$

$u'v + uv' = y^2 + 2xy\frac{dy}{dx}$

$3x^2y$

$u = 3x^2 \quad v = y$

$u' = 6x \quad v' = dy/dx$

$u'v + uv' = 6xy + 3x^2 dy/dx$

$x = 1$ find y?

$$xy^2 + 3x^2y = 4$$

$$y^2 + 3y = 4$$

$$y^2 + 3y - 4 = 0$$

$$(y+4)(y-1) = 0$$

$y = -4$ & $y = 1$ As $y > 0$, $y = 1$

$\therefore$ Pt $(1, 1)$ can now find m, gradient

$$m = \frac{dy}{dx} = \frac{-y(6x+y)}{x(3x+2y)} = \frac{-1(6+1)}{1(3+2)} = -\frac{7}{5}$$

$$(y - 1) = -\frac{7}{5}(x - 1)$$

$$5y - 5 = -7x + 7$$

$$7x + 5y - 12 = 0$$

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AH Implicit/Parametric Hwk 12

Q 11. $y = t^3 - \frac{5}{2}t^2$ | $x = \sqrt{t} = t^{1/2}$

$\frac{dy}{dt} = 3t^2 - 5t$
$= t(3t-5)$

$\frac{dx}{dt} = \frac{1}{2}t^{-1/2} = \frac{1}{2\sqrt{t}}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{t(3t-5)}{\left(\frac{1}{2\sqrt{t}}\right)} = 2\sqrt{t} \times t(3t-5)$$

$$= 2t^{3/2}(3t-5)$$

or $2\sqrt{t^3}\,(3t-5)$

4

$$d^2y/dx^2 = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(6t^{5/2} - 10t^{3/2}\right)}{dx/dt}$$

$$= \frac{15t^{3/2} - 15t^{1/2}}{\left(\frac{1}{2\sqrt{t}}\right)}$$

$$= 2t^{1/2}\left(15t^{3/2} - 15t^{1/2}\right)$$

$$\therefore \frac{d^2y}{dx^2} = 30t^2 - 30t$$

$$= at^2 + bt$$

$\therefore$ $a = 30$ & $b = -30$

3

Q11.

$$\frac{dy}{dx} = 2\sqrt{t^3}\,(3t-5)$$

At a point of Inflexion

$$\frac{d^2y}{dx^2} = 0$$

$$\frac{d^2y}{dx^2} = 30t^2 - 30t$$

$$\Rightarrow 30t(t-1) = 0$$

$30t = 0$ $\quad$ $t - 1 = 0$

$t = 0$ $\quad$ $t = 1$

But $t > 0 \Rightarrow t = 1$

$$\therefore \frac{dy}{dx} = 2\sqrt{t^3}\,(3t-5) = 2(\sqrt{1^3})(3-5) = 2(-2)$$

ie. $m = -4$

At $t = 1$:-

$x = \sqrt{t} = \sqrt{1} = 1$ $\qquad$ Pt $(1, -3/2)$

$y = t^3 - \frac{5}{2}t^2 = 1 - \frac{5}{2} = -3/2$

Hence Equation of tangent at $(1, -3/2)$, $m = -4$

$$(y + 3/2) = -4(x-1)$$

$$y + 3/2 = -4x + 4$$

$$2y + 3 = -8x + 8$$

$$8x + 2y - 5 = 0$$

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