Tues 9TH Sept : 2.20 Advanced Review

$\boxed{50}$ 1/10

Q79. a) $y(x) = x^2 e^{-x^3}$

$$\begin{aligned} u &= x^2 & v &= e^{-x^3} \\ u' &= 2x & v' &= -3x^2 e^{-x^3} \end{aligned}$$

$y'(x) = u'v + uv'$

$= 2xe^{-x^3} - 3x^4 e^{-x^3}$

$= \underline{\underline{xe^{-x^3}(2 - 3x^3)}}$

3

b) $g(x) = \dfrac{\sin x}{2 + \cos x}$ $\quad -\pi \leq x \leq \pi$

$$\begin{aligned} u &= \sin x & v &= 2 + \cos x \\ u' &= \cos x & v' &= -\sin x \end{aligned}$$

$g'(x) = \dfrac{u'v - uv'}{v^2}$

$= \dfrac{\cos x(2 + \cos x) - \sin x(-\sin x)}{(2 + \cos x)^2}$

$= \dfrac{2\cos x + \cos^2 x + \sin^2 x}{(2 + \cos x)^2}$

$\therefore g'(x) = \underline{\underline{\dfrac{2\cos x + 1}{(2 + \cos x)^2}}}$

3

c) $h(x) = \cos(x^2)\sin(3x)$

$$\begin{aligned} u &= \cos(x^2) & v &= \sin(3x) \\ u' &= -2x\sin(x^2) & v' &= 3\cos(3x) \end{aligned}$$

$h'(x) = u'v + uv'$

$= \underline{\underline{-2x\sin(x^2) \cdot \sin(3x) + 3\cos(3x)\cos(x^2)}}$

2

[Can't do anything else really
So this is enough]

2/10

Q79. (d) $f(x) = \frac{\ln|x+4|}{x+4}$, $x > -4$

$u = \ln|x+4|$ $v = x+4$
$u' = \frac{1}{(x+4)}$ $v' = 1$

$$f'(x) = \frac{u'v - uv'}{v^2}$$

$$= \frac{\frac{1}{(x+4)}.(x+4) - \ln|x+4|.1}{(x+4)^2}$$

$$= \frac{1 - \ln|x+4|}{(x+4)^2}$$

3

Q80 $f(x) = \frac{3x}{(x-2)}$, $x \neq 2$

$u = 3x$ $v = (x-2)$
$u' = 3$ $v' = 1$

$$f'(x) = \frac{u'v - uv'}{v^2}$$

$$= \frac{3(x-2) - 1.3x}{(x-2)^2}$$

3

$$= \frac{3x - 6 - 3x}{(x-2)^2}$$

$$= \frac{-6}{(x-2)^2}$$

As $(x-2)^2 > 0$ $\forall x \in R$, $x \neq 2$

Then $\frac{-6}{(x-2)^2} < 0$ $\forall x \in R$, $x \neq 2$

Thus $f'(x) < 0$ for all x, and is ALWAYS DECREASING

$$u = (1-x)^2 \qquad v = e^x$$
$$u' = 2(1-x)^1 \cdot -1 \qquad v' = e^x$$
$$= -2(1-x)$$

Q8 1. $f(x) = (1-x)^2 e^x$

$f'(x) = u'v + uv'$

$$= -2(1-x)e^x + (1-x)^2 e^x$$
$$= (1-x)e^x\left[-2+(1-x)\right]$$
$$= (1-x)e^x\left[-2+1-x\right]$$
$$= (1-x)e^x(-1-x)$$
$$= -(1+x)(1-x)e^x \quad \text{(or alternative)}$$

Stat Pts to determine $\boxed{\text{Nature}}$

$f'(x) = 0 \Rightarrow -(1+x)(1-x)e^x = 0$

$-(1+x) = 0 \qquad 1-x=0 \qquad e^x = 0$

$x = -1 \qquad x = 1 \qquad$ × Impossible!

	x	$\xrightarrow{(-2)}$	-1	$\xrightarrow{(0)}$	1	$\xrightarrow{(2)}$
$f'(x) =$	$-(1+x)$	+	0	−	−	−
	$(1-x)$	+	+	+	0	−
	e^x	+	+	+	+	+
	$f'(x)$	+	0	−	0	+
	Slope	/	—	\	—	/

Thus $f(x)$ is decreasing $-1 < x < 1$

6

Q82. $y = e^x \sin x \quad , 0 < x < \pi$

$$\frac{dy}{dx} = u'v + uv'$$

$$\boxed{\begin{array}{ll} u = e^x & v = \sin x \\ u' = e^x & v' = \cos x \end{array}}$$

$$= e^x \sin x + e^x \cos x$$

$$= e^x(\sin x + \cos x)$$

3

When $\frac{dy}{dx} = 0$: $e^x(\sin x + \cos x) = 0$

$e^x = 0$

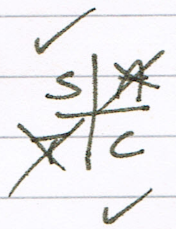
Impossible! ✗

$\sin x + \cos x = 0$

$\sin x = -\cos x$

$$\frac{\sin x}{\cos x} = \frac{-\cos x}{\cos x}$$

$\tan x = -1$



1st Quad $x = \pi/4$

2nd: $\pi - \pi/4 = 3\pi/4$

4th : $2\pi - \pi/4 = \frac{7\pi}{4}$

But only asked in range $0 < x < \pi$

$$\therefore \quad x = \frac{3\pi}{4}$$

3

Q83 $y = \dfrac{\cos(kx)}{x}$

$$\boxed{\begin{array}{ll} u = \cos(kx) & v = x \\ u' = -k\sin(kx) & v' = 1 \end{array}}$$

3

$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{-kx\sin(kx) - \cos(kx)}{x^2}$$

$$\boxed{-kx\sin(kx)} \quad \begin{array}{ll} a = -kx & b = \sin(kx) \\ a' = -k & b' = k\cos(kx) \end{array}$$

a b

Then $a'b + ab' = -k\sin(kx) - k^2x\cos(kx)$

$(-kx\sin(kx))$ is product rule 'nested' within a quotient rule problem $(u'v+uv')$ or as above $(a'b+ab')$ needs done first

$$\begin{aligned} u &= (-kx\sin(kx)) - \cos(kx) \\ u' &= -k\sin(kx) - k^2x\cos(kx) + k\sin(kx) \\ &= -k^2x\cos(kx) \\ v &= x^2 \\ v' &= 2x \end{aligned}$$

$$\frac{d^2y}{dx^2} = \frac{(-k^2x\cos(kx))x^2 - 2x\left[(-kx\sin(kx) - \cos(kx))\right]}{(x^2)^2}$$

$$= \frac{-k^2x^3\cos(kx) + 2kx^2\sin(kx) + 2x\cos(kx)}{x^4}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{-k^2x^2\cos(kx) + 2kx\sin(kx) + 2\cos(kx)}{x^3}$$

3

Q3. OR for $\frac{d^2y}{dx^2}$ Separate $\frac{dy}{dx}$ into 2 problems

$$\frac{dy}{dx} = \frac{-kx\sin(kx) - \cos(kx)}{x^2} = \frac{-k\sin(kx)}{x} + \frac{(-\cos(kx))}{x^2}$$

$$\frac{-k\sin(kx)}{x}$$

$u = -k\sin(kx)$
$u' = -k^2\cos(kx)$
$v = x$
$v' = 1$

$$\frac{u'v - uv'}{v^2}$$

$$= \frac{-k^2x\cos(kx) + k\sin(kx)}{x^2}$$

$$= \frac{-k^2x^2\cos(kx) + kx\sin(kx)}{x^3}$$

$$\frac{-\cos(kx)}{x^2}$$

$u = -\cos(kx)$
$u' = k\sin(kx)$
$v = x^2$
$v' = 2x$

$$\frac{u'v - uv'}{v^2}$$

$$= \frac{kx^2\sin(kx) + 2x\cos(kx)}{x^4}$$

$$= \frac{kx\sin(kx) + 2\cos(kx)}{x^3}$$

$$\frac{d^2y}{dx^2} = \frac{-k^2x^2\cos(kx) + kx\sin(kx) + kx\sin(kx) + 2\cos(kx)}{x^3}$$

$$= \frac{-k^2x^2\cos(kx) + 2kx\sin(kx) + 2\cos(kx)}{x^3}$$

(*) Either method produces $\frac{d^2y}{dx^2}$ so decide which method works best for you

Now can attempt to solve problem →

7/10

$$\boxed{\frac{d^2y}{dx^2} + \frac{2}{x}\left(\frac{dy}{dx}\right) + k^2y = 0}$$

Q83.

$$\left(\frac{-k^2x^2\cos(kx) + 2kx\sin(kx) + 2\cos(kx)}{x^3}\right) + \frac{2}{x}\left(\frac{-kx\sin(kx) - \cos(kx)}{x^2}\right) + k^2\left(\frac{\cos(kx)}{x}\right)$$

$$= \frac{-k^2x^2\cos(kx) + 2kx\sin(kx) + 2\cos(kx)}{x^3} + \left(\frac{-2kx\sin(kx) - 2\cos(kx)}{x^3}\right) + \frac{k^2\cos(kx)}{x}$$

$$= \frac{\cancel{-k^2x^2}\cos(kx) + \cancel{2kx\sin}(kx) + 2\cancel{\cos}(kx) - \cancel{2kx\sin}(kx) - 2\cancel{\cos}(kx) + \cancel{k^2x^2}\cos(kx)}{x^3}$$

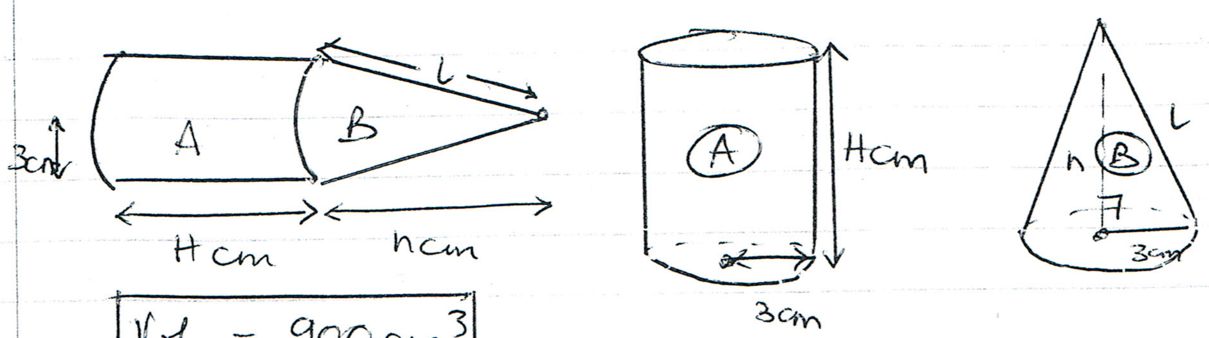
$$= \frac{0}{x^3}$$

$= \underline{0}$ as required

3

084.

8/10



$\boxed{\text{Vol} = 900\,\text{cm}^3}$

$$\text{Vol } A = \pi r^2 h = \pi \times 3^2 \times H = \underline{\underline{9\pi H}}\ \text{cm}^3$$

$$\text{Vol } B = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 3^2 \times h = \underline{\underline{3\pi h}}\ \text{cm}^3$$

(a) Total Volume, $V = 900\text{cm}^3$

Thus

$$V_A + V_B = 900$$

$$9\pi H + 3\pi h = 900$$

$$3\pi(3H + h) = 900$$

$$3H + h = \frac{900}{3\pi}$$

$$3H = \frac{300}{\pi} - h$$

$$\underline{\underline{H = \frac{1}{3}\left(\frac{300}{\pi} - h\right)}} \text{ or } \left(\frac{100}{\pi} - \frac{h}{3}\right)$$

3

(b) SA cylinder = Base Area + Curved SA.

$$= \pi r^2 + \pi D H$$

$$= \pi \times 3^2 + \pi \times (3 \times 2) \times H$$

$$= 9\pi + 6\pi H$$

$$= 9\pi + 6\pi\left(\frac{1}{3}\left(\frac{300}{\pi} - h\right)\right)$$

$$= 9\pi + 2\pi\left(\frac{300}{\pi} - h\right)$$

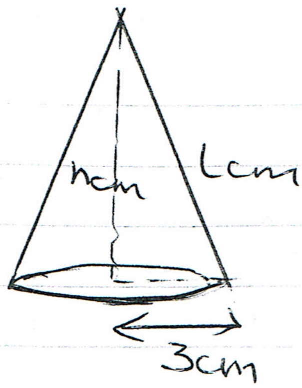
3

$$= \underline{\underline{9\pi + 600 - 2\pi h}}\ \text{cm}^2$$

10/10

Q84 (cont...)

(b)



$L^2 = h^2 + 3^2$

$= h^2 + 9$

$L = \sqrt{h^2+9}$ cm

$SA_{cone} = \pi r L$

$= \pi \times 3 \times \sqrt{h^2+9}$

$= 3\pi \sqrt{h^2+9}$ cm²

3

Total SA = $SA_{cyl} + SA_{cone}$

$= (9\pi + 600 - 2\pi h) + (3\pi \sqrt{h^2+9})$

$= 600 - 2\pi h + 9\pi + 3\pi\sqrt{h^2+9}$ cm²

as required.

9/10

Q84

(c) $SA = 600 + 9\pi - 2\pi h + 3\pi\sqrt{h^2+9}$

$= 600 + 9\pi - 2\pi h + 3\pi(h^2+9)^{1/2}$

$$\frac{d(SA)}{dh} = -2\pi + \frac{1}{\cancel{2}}\cdot\left(3\pi(h^2+9)^{-1/2}\right)\cdot\cancel{2}h$$

$$= -2\pi + 3\pi h(h^2+9)^{-1/2}$$

Min / Max $\frac{d(SA)}{dh} = 0$

$$= -2\pi + \frac{3\pi h}{\sqrt{h^2+9}}$$

$$-2\pi + \frac{3\pi h}{\sqrt{h^2+9}} = 0$$

$$\frac{3\pi h}{\sqrt{h^2+9}} = 2\pi$$

$$\frac{3h}{\sqrt{h^2+9}} = 2$$

(Squaring) $$\frac{9h^2}{h^2+9} = 4$$ 6

$$9h^2 = 4h^2 + 36$$

$$5h^2 = 36$$

$$h^2 = 7.2$$

$$h = 2.683 \quad (\text{As } h > 0)$$

x	$\xrightarrow{(2)}$	$\sqrt{7.2}$	$\xrightarrow{(3)}$
$\frac{d(SA)}{dh}$	$-$	0	$+$
Slope	\	—	/

$\therefore$ Min at $h = 2.68$ cm (to 3sf)

$\therefore$ $h = 2.68$ cm (to 3sf) gives a minimum Surface Area