AH 2008/9 Prelim (Units 1/2)

Q1.

$$x = \sin^{-1}(t)$$

$$\frac{dx}{dt} = \frac{1}{\sqrt{1-t^2}}$$

$$y = \ln t$$

$$\frac{dy}{dt} = \frac{1}{t}$$

$$dy/dx = \frac{dy/dt}{dx/dt}$$

$$= \frac{\frac{1}{t}}{\frac{1}{\sqrt{1-t^2}}}$$

$$= \frac{1}{t} \div \frac{1}{\sqrt{1-t^2}}$$

$$= \frac{1}{t} \times \frac{\sqrt{1-t^2}}{1}$$

$$\therefore \frac{dy}{dx} = \frac{\sqrt{1-t^2}}{t}$$

- Use parametric Diff
- Diff correctly
- Simplify

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Q2.

$$\boxed{(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} (x)^{n-r} (y)^r}$$

To find co-efficients need to separate constants and terms of x

$$\left(\frac{x^2}{2} - \frac{4}{x^3}\right)^8 = \sum_{r=0}^{8} \binom{8}{r} \left(\frac{x^2}{2}\right)^{8-r} \left(\frac{-4}{x^3}\right)^r$$

So each term is: $\binom{8}{r}\left(\frac{1}{2}\right)^{8-r}(x^2)^{8-r}(-4)^r\left(\frac{1}{x^3}\right)^r$

$$= \binom{8}{r}\left(\frac{1}{2}\right)^{8-r}(-4)^r \times (x^2)^{8-r} \times (x^{-3})^r$$

Coefficient, $C = \binom{8}{r}\left(\frac{1}{2}\right)^{8-r}(-4)^r$

$$= \binom{8}{4}\left(\frac{1}{2}\right)^{8-4}(-4)^4$$

$$C = \frac{8!}{4!4!} \times \left(\frac{1}{2}\right)^4 \times (-4)^4$$

$$= \frac{\cancel{8}^{2}\times 7\times \cancel{6}\times 5\times \cancel{4!}}{\cancel{4!}\times(\cancel{4}\times\cancel{3}\times\cancel{2}\times 1)} \times \frac{1}{\cancel{16}_{1}} \times \cancel{256}^{16}$$

$$\therefore C = 70 \times 16 = \underline{1120}$$

For x^{-4}:

$$x^{16-2r} \times x^{-3r} = x^{-4}$$

$$x^{16-5r} = x^{-4}$$

$$\therefore 16 - 5r = -4$$

$$-5r = -20$$

$$\therefore \underline{r = 4}$$

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Alternative Q2 Solution

$$\left(\frac{x^2}{2}-\frac{4}{x^3}\right)^8 = \binom{8}{0}\left(\frac{x^2}{2}\right)^8\left(\frac{-4}{x^3}\right)^0 + \binom{8}{1}\left(\frac{x^2}{2}\right)^7\left(\frac{-4}{x^3}\right)^1 + \binom{8}{2}\left(\frac{x^2}{2}\right)^6\left(\frac{-4}{x^3}\right)^2 + \binom{8}{3}\left(\frac{x^2}{2}\right)^5\left(\frac{-4}{x^3}\right)^3 + \binom{8}{4}\left(\frac{x^2}{2}\right)^4\left(\frac{-4}{x^3}\right)^4$$

$$+ \binom{8}{5}\left(\frac{x^2}{2}\right)^3\left(\frac{-4}{x^3}\right)^5 + \binom{8}{6}\left(\frac{x^2}{2}\right)^2\left(\frac{-4}{x^3}\right)^6 + \binom{8}{7}\left(\frac{x^2}{2}\right)^1\left(\frac{-4}{x^3}\right)^7 + \binom{8}{8}\left(\frac{x^2}{2}\right)^0\left(\frac{-4}{x^3}\right)^8$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
$\boxed{*\ 1\ 8\ 28\ 56\ 70\ 56\ 28\ 8\ 1}$

$$= (1)\left(\frac{x^{16}}{256}\right)(1) + (8)\left(\frac{x^{14}}{128}\right)\left(\frac{-4}{x^3}\right) + (28)\left(\frac{x^{12}}{64}\right)\left(\frac{16}{x^6}\right) + (56)\left(\frac{x^{10}}{32}\right)\left(-\frac{64}{x^9}\right) + (70)\left(\frac{x^8}{16}\right)\left(\frac{256}{x^{12}}\right)$$

$$+ (56)\left(\frac{x^6}{8}\right)\left(-\frac{1024}{x^{15}}\right) + (28)\left(\frac{x^4}{4}\right)\left(\frac{4096}{x^{18}}\right) + (8)\left(\frac{x^2}{2}\right)\left(\frac{-16384}{x^{21}}\right) + (1)(1)\left(\frac{65536}{x^{24}}\right)$$

$$\therefore \left(\frac{x^2}{2}-\frac{4}{x^3}\right)^8 = \frac{x^{16}}{256} - \frac{x^{11}}{4} + 7x^6 - 112x + \frac{1120}{x^4} - \frac{7168}{x^9} + \frac{28672}{x^{14}} - \frac{65536}{x^{19}} + \frac{65536}{x^{24}}$$

$$\boxed{\frac{1120}{x^4} = 1120x^{-4}}$$

Thus Coefficient of x^{-4} = $\underline{\underline{1120}}$ •

$\ominus$ for eased approach

- Expands with correct powers
- Correct Coeffs $\binom{n}{r}$
- Completes each term
- 1120

Q3. $\dfrac{4}{x^3-4x^2} = \dfrac{4}{x^2(x-4)} = \dfrac{A}{(x-4)} + \dfrac{B}{x} + \dfrac{C}{x^2}$

$\therefore \quad Ax^2 + Bx(x-4) + C(x-4) = 4$

(Alternative below)

<u>Let $x=4$</u> :

$A(4)^2 + 0 + 0 = 4$

$16A = 4$

$\therefore \quad A = 1/4$

<u>Let $x=0$</u>:

$0 + 0 + C(0-4) = 4$

$-4C = 4$

$\therefore \quad C = -1$

1

<u>Let $x=1$</u> :

$A(1)^2 + B(1-4) + C(1-4) = 4$

$A - 3B - 3C = 4$

$\frac{1}{4} - 3B + 3 = 4$

$-3B = 4 - 3\frac{1}{4}$

$-3B = \frac{3}{4}$

$B = \frac{-1}{4}$

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$\therefore \quad \dfrac{4}{x^3-4x^2} = \dfrac{\left(\frac{1}{4}\right)}{(x-4)} + \dfrac{\left(\frac{-1}{4}\right)}{x} + \dfrac{(-1)}{x^2} = \dfrac{1}{4(x-4)} - \dfrac{1}{4x} - \dfrac{1}{x^2}$

* Recall repeated factors (need to go up in powers)
* As x^2 is quadratic, could also use technique for quadratic problems ie. $\left[\dfrac{A}{(x-4)} + \dfrac{(Bx+C)}{x^2}\right]$, but not nec.

Q3 Alternative 'Quadratic Approach'

$$\frac{4}{x^3-4x^2} = \frac{4}{x^2(x-4)} = \frac{A}{(x-4)} + \frac{(Bx+C)}{x^2}$$

$$\therefore \quad Ax^2 + (Bx+C)(x-4) = 4$$

Let $x=4$:
$$A(4)^2 + 0 = 4$$
$$16A = 4$$
$$\therefore \quad A = 1/4$$

Let $x=0$:
$$0 + (0+C)(0-4) = 4$$
$$-4C = 4$$
$$\therefore \quad C = -1$$

Let $x=1$:
$$A + (B+C)(1-4) = 4$$
$$\frac{1}{4} + (B-1)(-3) = 4$$
$$\frac{1}{4} + 3 - 3B = 4$$
$$-3B = 3/4$$
$$-B = 1/4$$
$$\therefore \quad B = -1/4$$

$$\therefore \quad \frac{A}{(x-4)} + \frac{(Bx+C)}{x^2} = \frac{1/4}{(x-4)} + \frac{(-1/4x-1)}{x^2}$$

$$= \frac{1}{4(x-4)} - \frac{\frac{1}{4}x}{x^2} - \frac{1}{x^2}$$

$$= \frac{1}{4(x-4)} - \frac{1}{4x} - \frac{1}{x^2}$$

(4)

[Which is the same as the repeated factors solution, but more awkward]

Q3.

$$\int \frac{4}{x^3-4x^2}\,dx = \int \left(\frac{1}{4(x-4)} - \frac{1}{4x} - \frac{1}{x^2}\right)dx$$

$$= \int \frac{dx}{4(x-4)} - \frac{1}{4}\int \frac{dx}{x} - \int \frac{dx}{x^2}$$

$$= \frac{1}{4}\int \frac{dx}{(x-4)} - \frac{1}{4}\int \frac{dx}{x} - \int x^{-2}\,dx$$

$$= \frac{1}{4}\ln|x-4| - \frac{1}{4}\ln|x| - \frac{x^{-1}}{-1} + c$$

$$= \frac{1}{4}\ln\left|\frac{x-4}{x}\right| + \frac{1}{x} + c$$

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OR $$\ln\left|\left(\frac{x-4}{x}\right)^{1/4}\right| + \frac{1}{x} + c$$

Q4.

$$\begin{aligned} x + y - 3z &= 0 \\ 3x - y - 5z &= 4 \\ x \quad\quad - 2z &= 1 \end{aligned} \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & -3 & 0 \\ 3 & -1 & -5 & 4 \\ 1 & 0 & -2 & 1 \end{array}\right)\begin{array}{l} r_1 \\ r_2 \\ r_3 \end{array} \rightarrow \begin{array}{r} 3r_1 \\ r_2 \\ 3r_3 \end{array}\left(\begin{array}{ccc|c} 3 & 3 & -9 & 0 \\ 3 & -1 & -5 & 4 \\ 3 & 0 & -6 & 3 \end{array}\right)\begin{array}{l} r_1 \\ r_2 \\ r_3 \end{array}$$

$$\begin{array}{r} r_1 \\ r_2-r_1 \\ r_3-r_1 \end{array}\left(\begin{array}{ccc|c} 3 & 3 & -9 & 0 \\ 0 & -4 & 4 & 4 \\ 0 & -3 & 3 & 3 \end{array}\right)\begin{array}{l} r_1 \\ r_2 \\ r_3 \end{array} \rightarrow \begin{array}{r} \frac{1}{3}r_1 \\ \frac{1}{4}r_2 \\ \frac{1}{3}r_3 \end{array}\left(\begin{array}{ccc|c} 1 & 1 & -3 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 1 \end{array}\right)\begin{array}{l} r_1 \\ r_2 \\ r_3 \end{array} \rightarrow \begin{array}{r} r_1 \\ r_2 \\ r_3-r_2 \end{array}\left(\begin{array}{ccc|c} 1 & 1 & -3 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)\begin{array}{l} r_1 \\ r_2 \\ r_3 \end{array}$$

As $0x + 0y + 0z = 0 \Rightarrow$ Infinite solutions exist.
this is REDUNDANT

Let $z = z$; $-y + z = 1$; $x + y - 3z = 0$

$-y = 1 - z$ $\qquad x = 3z - y$

$y = z - 1$ $\qquad = 3z - (z-1)$

$\therefore x = 2z + 1$

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[In some text, set $z = t$ or $z = k$ & rearrange to represent x & y in terms of this] (If $z = t$; $y = t - 1$ & $z = 2t + 1$)

Q5. $z = \frac{5i}{1+2i} = \frac{5i}{(1+2i)} \times \frac{(1-2i)}{(1-2i)} = \frac{5i-10i^2}{1-2i+2i-4i^2} = \frac{5i+10}{1+4}$

$z = \frac{5i+10}{5}$

$z = i+2$

$\therefore \; z = 2+i$

Find z, z^2, z^3 & z^4 first:

$z^1 = 2+i$

$z^2 = (2+i)^2 = (4+2i+2i+i^2) = 4+4i-1 = 3+4i$

$z^3 = (2+i)(3+4i) = 6+8i+3i+4i^2 = 6+11i-4 = 2+11i$

$z^4 = (2+i)(2+11i) = 4+22i+2i+11i^2 = 4+24i-11 = -7+24i$

$z^4 - 4z^3 + 6z^2 - 4z + 5 = 0$

$(-7+24i) - 4(2+11i) + 6(3+4i) - 4(2+i) + 5$

$= -7+24i - 8 - 44i + 18 + 24i - 8 - 4i + 5$

$= (-7-8+18-8+5) + i(24-44+24-4)$

$= \underline{\underline{0}} \quad \therefore \; z = 2+i$ is a solution

Alternatively use synthetic division, but awkward

$z = 2+i$

$$\begin{array}{c|ccccc} (2+i) & 1 & -4 & 6 & -4 & 5 \\ & \downarrow & (2+i) & -5 & (2+i) & -5 \\ \hline & 1 & (-2+i) & 1 & (-2+i) & \underline{\underline{0}} \end{array}$$

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$\therefore \; z = 2+i$ is a ~~factor~~ / solution as $R = 0$

(OR)

Q5. $z = (2+i) \Rightarrow \bar{z} = (2-i)$

By the fundamental theorem of algebra a CONJUGATE PAIR exists

* Using 2 solutions $z_1 = (2+i)$ & $z_2 = (2-i)$
* Can then find 2 factors of Quartic: $(z_1 - 2 - i) = 0$ & $(z_2 - 2 + i) = 0$
* Now if we multiply 2 factors ⇒ Obtain a Quadratic
  ⇒ Use long division
  ⇒ Find remaining 2 sols

$$(z-2-i)(z-2+i) = z^2 - 4z + 4 - i^2$$
$$= z^2 - 4z + 4 + 1$$
$$= z^2 - 4z + 5$$

	z	-2	$-i$
z	z^2	$-2z$	$\cancel{-iz}$
-2	$-2z$	$+4$	$\cancel{+2i}$
$+i$	$\cancel{+iz}$	$\cancel{-2i}$	$-i^2$

Now by dividing shall be able to find remainder and solve 4 solutions for $z^4 - 4z^3 + 6z^2 - 4z + 5 = 0$

$$\begin{array}{r|l} & z^2 + 1 \\ \hline z^2 - 4z + 5 & z^4 - 4z^3 + 6z^2 - 4z + 5 \\ & z^4 - 4z^3 + 5z^2 \\ \hline & \qquad\qquad z^2 - 4z + 5 \\ & \qquad\qquad z^2 - 4z + 5 \\ \hline & \qquad\qquad\qquad - \end{array}$$

$\therefore z^2 + 1 = 0$

$z^2 = -1$

$z^2 = i^2$

$z = \pm i$

③

$\therefore$ 4 Solutions are:

$z_1 = 2+i$; $z_2 = 2-i$; $z_3 = i$; $z_4 = -i$

Q6. $\boxed{U_n = ar^{n-1}}$ $\boxed{S_\infty = \dfrac{a}{1-r}}$ So need to find a & r.

$$\boxed{\begin{aligned} U_2 &= ar^{2-1} = ar = 1/6 \\ U_5 &= ar^{5-1} = ar^4 = 1/48 \end{aligned}}$$

For Geometric problems use ratio/proportion to solve.

$$\frac{\cancel{a}r^4}{\cancel{a}r} = \frac{\frac{1}{48}}{1/6}$$

$$r^3 = 6/48$$

$$r^3 = 1/8$$

$$\therefore \underline{\underline{r = 1/2}}$$

If $r = 1/2$

$$U_2 = ar = 1/6$$

$$\tfrac{1}{2}a = 1/6$$

$$a = 2/6$$

$$\therefore \underline{\underline{a = 1/3}}$$

$$S_\infty = \frac{a}{1-r} = \frac{1/3}{1-1/2} = \frac{1/3}{1/2} = \underline{\underline{\frac{2}{3}}}$$

As $a = \frac{1}{3}$ $S_\infty = \frac{2}{3}$ $\Rightarrow$ $S_\infty = 2a = 2u_1$

4

Q7.

$$x^3y^2 - 2xy + 8 = 0$$
① ②

$$3x^2y^2 + 2x^3y\frac{dy}{dx} - 2y - 2x\frac{dy}{dx} = 0$$

$$(2x^3y - 2x)\frac{dy}{dx} = 2y - 3x^2y^2$$

$$\therefore \frac{dy}{dx} = \frac{2y - 3x^2y^2}{2x^3y - 2x}$$

OR $$\frac{dy}{dx} = \frac{y(2 - 3x^2y)}{2x(x^2y - 1)}$$

① x^3y^2

$$\boxed{\begin{matrix} u = x^3 & v = y^2 \\ u' = 3x^2 & v' = 2y\frac{dy}{dx} \end{matrix}}$$

$u'v + uv'$

$= 3x^2y^2 + 2x^3y\frac{dy}{dx}$

② $-2xy$

$$\boxed{\begin{matrix} u = -2x & v = y \\ u' = -2 & v' = \frac{dy}{dx} \end{matrix}}$$

$u'v + uv'$

$= -2y - 2x\frac{dy}{dx}$

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$x = -1$ (Find y from original)

$$x^3y^2 - 2xy + 8 = 0$$
$$(-1)^3y^2 - 2(-1)y + 8 = 0$$
$$-y^2 + 2y + 8 = 0$$
$$y^2 - 2y - 8 = 0$$
$$(y+2)(y-4) = 0$$
$$y = -2 \quad y = 4$$

But $y < 0$, $\therefore y = -2$

Pt $(-1, -2)$

Gradient + Equation

$$m = \frac{dy}{dx} = \frac{2y - 3x^2y^2}{2x^3y - 2x}$$

$$= \frac{2(-2) - 3(-1)^2(-2)^2}{2(-1)^3(-2) - 2(-1)}$$

$$= \frac{-4 - 12}{4 + 2} = \frac{-16}{6}$$

$$\therefore m = -\frac{8}{3}$$

$$(y+2) = -\frac{8}{3}(x+1)$$

$$3y + 6 = -8x - 8$$

$$8x + 3y + 14 = 0$$ is Eqⁿ of Tangent.

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Q8. $y = (x+1)^{x^2}$

As 'Powers' involved
$\Rightarrow$ MUST USE LOGARITHMIC DIFF

$\ln(y) = \ln\left((x+1)^{x^2}\right)$

$\ln y = x^2 \ln(x+1)$

$\frac{1}{y}\frac{dy}{dx} = 2x \ln|x+1| + \frac{x^2}{(x+1)}$

Let $u = x^2$ $v = \ln(x+1)$
$u' = 2x$ $v' = \frac{1}{x+1}$
$u'v + uv'$
$= 2x\ln|x+1| + \frac{x^2}{(x+1)}$

$\therefore \frac{dy}{dx} = \left(2x \ln|x+1| + \frac{x^2}{(x+1)}\right) \times y$

$\frac{dy}{dx} = \left(2x \ln|x+1| + \frac{x^2}{(x+1)}\right) \times (x+1)^{x^2}$ 4

May rearrange further, but not necessary

$\frac{dy}{dx} = \left(\frac{(x+1)2x\ln|x+1| + x^2}{(x+1)}\right) \times (x+1)^{x^2}$

$= x\left(2(x+1)\ln|x+1| + x\right) \times \frac{(x+1)^{x^2}}{(x+1)}$

$= x\left(2(x+1)\ln|x+1| + x\right)(x+1)^{x^2-1}$

Q9. $I = \int_0^2 e^{\sqrt{4x+1}}\,dx$

Let $u = \sqrt{4x+1} = (4x+1)^{1/2}$

$\frac{du}{dx} = \frac{1}{2}(4x+1)^{-1/2}.4$

$\therefore \frac{du}{dx} = \frac{2}{\sqrt{4x+1}}$

So $\left(\frac{\sqrt{4x+1}}{2}\,du\right) = dx$

When $x=0$

$u = \sqrt{4(0)+1} = \sqrt{1} = \underline{\underline{1}}$

When $x=2$

$u = \sqrt{4(2)+1} = \sqrt{9} = \underline{\underline{3}}$

$= \int_1^3 e^u . \left(\frac{\sqrt{4x+1}}{2}.du\right)$

$= \int_1^3 e^u . \frac{u}{2}\,du$

$\therefore I = \frac{1}{2}\int_1^3 ue^u\,du$

Where $a=1$; $b=3$ & $k=2$

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(b) $I = \frac{1}{2}\int_1^3 ue^u\,du$

$\boxed{\text{Int. by Parts } \int uv' = uv - \int u'v}$

$\boxed{\begin{array}{ll} \text{Let } u = u & v = e^u \\ u' = 1 & v' = e^u \end{array}}$

$I = \frac{1}{2}\int_1^3 ue^u\,du$

$= \frac{1}{2}\left[\left[ue^u\right]_1^3 - \int_1^3 e^u\,du\right]$

$= \frac{1}{2}\left[(3e^3 - e^1) - \left[e^u\right]_1^3\right]$

$= \frac{1}{2}\left[(3e^3 - e^1) - (e^3 - e^1)\right]$

$= \frac{1}{2}\left(3e^3 - \cancel{e^1} - e^3 + \cancel{e^1}\right)$

$= \frac{1}{2}(2e^3)$

$= \underline{\underline{e^3}}$

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AH 2008/2 Prelim (Unit 1/2)

Q10. $f(x) = -x - \frac{4}{(x+1)}$

$x=0$: $y = 0 - \frac{4}{(0+1)} = \frac{-4}{1} = -4 \Rightarrow (0,-4)$

$y=0$: $0 = -x - \frac{4}{(x+1)}$

$x = \frac{-4}{(x+1)}$

$x^2 + x = -4$

$\therefore x^2 + x + 4 = 0$

$a=1$, $b=1$, $c=4$

$b^2-4ac = (1)^2 - (4 \times 1 \times 4) = 1-16 = -15$ $\therefore$ No Real Roots

As $b^2-4ac<0 \Rightarrow$ Does Not Cut x-axis

(3)

(b) Vertical Asymptote when $x+1=0$

$x=-1$

Non-Vertical/Oblique $y=-x$

(2)

(c) $y = -x - 4(x+1)^{-1}$

$\frac{dy}{dx} = -1 + 4(x+1)^{-2} = -1 + \frac{4}{(x+1)^2}$

Stat Pts $dy/dx = 0$

$-1 + \frac{4}{(x+1)^2} = 0$

$\frac{4}{(x+1)^2} = 1$

$4 = (x+1)^2$

$\therefore (x+1) = \pm 2$

$x = -1 \pm 2$

$\therefore x = -3$ & $x = 1$

$x=1$: $y = -1 - \frac{4}{(1+1)} = -1 - \frac{4}{2} = -3$

$\therefore (1,-3)$

$x=-3$: $y = -(-3) - \frac{4}{(-3+1)}$

$= 3 - \frac{4}{(-2)}$

$= 3+2$

$= 5$ $\therefore (-3,5)$

Q10(c)

$$y = -x - 4(x+1)^{-1}$$

$$\frac{dy}{dx} = -1 + 4(x+1)^{-2}$$

$$\frac{d^2y}{dx^2} = -8(x+1)^{-3} = \frac{-8}{(x+1)^3}$$

At $(1,-3)$ $\frac{d^2y}{dx^2} = \frac{-8}{(1+1)^3} = \frac{-8}{8} = -1$, As $\frac{d^2y}{dx^2} < 0 \Rightarrow \cap$ Max $(1,-3)$

At $(-3,5)$ $\frac{d^2y}{dx^2} = \frac{-8}{(-3+1)^3} = \frac{-8}{-8} = 1$, As $\frac{d^2y}{dx^2} > 0 \Rightarrow \cup$ Mini $(-3,5)$

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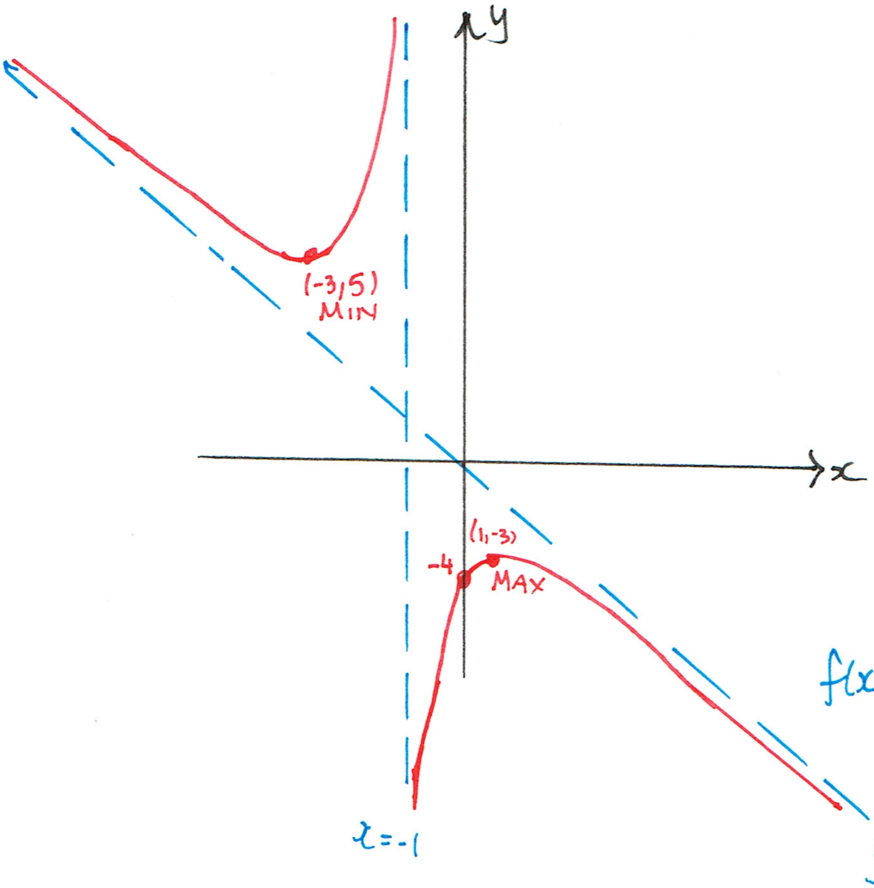
Q10(d) [Not required to sketch]



$|f(x)| \Rightarrow$ |MODULUS| gives POSITIVE values for $f(x)$

$\therefore$ Stat Pts are

Min $(-3,5)$
& Min $(1,3)$

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Q11. $z = \cos\theta + i\sin\theta$

a) $z^3 = (\cos\theta + i\sin\theta)^3 = \cos(3\theta) + i\sin(3\theta)$ 1

b) $(\cos\theta + i\sin\theta)^3$

$= \binom{3}{0}(\cos\theta)^3(i\sin\theta)^0 + \binom{3}{1}(\cos\theta)^2(i\sin\theta)^1 + \binom{3}{2}(\cos\theta)^1(i\sin\theta)^2 + \binom{3}{3}(\cos\theta)^0(i\sin\theta)^3$

$= (1)(\cos^3\theta)(1) + (3)(\cos^2\theta)(i\sin\theta) + (3)(\cos\theta)(i^2\sin^2\theta) + (1)(1)(i^3\sin^3\theta)$

$= \cos^3\theta + i3\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta$

$= (\cos^3\theta - 3\cos\theta\sin^2\theta) + i(3\cos^2\theta\sin\theta - \sin^3\theta)$ 2

(c) As $(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$

$= (\cos^3\theta - 3\cos\theta\sin^2\theta) + i(3\cos^2\theta\sin\theta - \sin^3\theta)$

$\therefore \cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$

$= \cos^3\theta - 3\cos\theta(1 - \cos^2\theta)$

$= \cos^3\theta - 3\cos\theta + 3\cos^3\theta$ 2

$= 4\cos^3\theta - 3\cos\theta$

& $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$

$= 3(1 - \sin^2\theta)\sin\theta - \sin^3\theta$

$= (3 - 3\sin^2\theta)\sin\theta - \sin^3\theta$

$= 3\sin\theta - 3\sin^3\theta - \sin^3\theta$

$\therefore \sin 3\theta = 3\sin\theta - 4\sin^3\theta$ 2

$$\boxed{\cos 3\theta = 4\cos^3\theta - 3\cos\theta \quad \& \quad \sin 3\theta = 3\sin\theta - 4\sin^3\theta}$$

$$\cot 3\theta = \frac{1}{\tan 3\theta} = \frac{\cos 3\theta}{\sin 3\theta} = \frac{4\cos^3\theta - 3\cos\theta}{3\sin\theta - 4\sin^3\theta}$$

$(\div \text{ by } \cos^3\theta)$

$$= \frac{\left(4 - 3/\cos^2\theta\right)}{\left(3\frac{\sin\theta}{\cos^3\theta} - 4\frac{\sin^3\theta}{\cos^3\theta}\right)}$$

Recall

$1 + \tan^2\theta = \sec^2\theta$

$$= \frac{4 - 3\sec^2\theta}{3\tan\theta\sec^2\theta - 4\tan^3\theta}$$

$$= \frac{4 - 3(1 + \tan^2\theta)}{3\tan\theta(1 + \tan^2\theta) - 4\tan^3\theta}$$

$$= \frac{4 - 3 - 3\tan^2\theta}{3\tan\theta + 3\tan^3\theta - 4\tan^3\theta}$$

$$\therefore \quad \cot 3\theta = \frac{1 - 3\tan^2\theta}{3\tan\theta - \tan^3\theta}$$

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As required

AH 2008/9 Prelim (Unit 1/2)

Q12. a) $\dfrac{x+8}{x-1} = 1 + \dfrac{9}{(x-1)}$

$$x-1 \overline{\big)\, x+8} \quad \begin{array}{r} 1 \\ x+8 \\ \underline{x-1} \\ \underline{\underline{9}} \end{array}$$

$\therefore$ $A=1$ & $B=9$

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b) $y=y$

$$\frac{x+8}{x-1} = 12-x$$

$$x+8 = (12-x)(x-1)$$

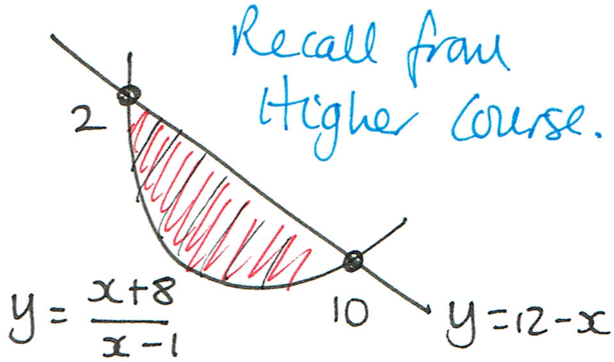
$$x+8 = 12x-12-x^2+x$$

$$x^2-12x+20=0$$

$$(x-2)(x-10)=0$$

$$x=2 \quad \& \quad x=10$$

Recall from Higher course.



$$\int_2^{10} (\text{Above} - \text{Below})\,dx$$

$$\text{Area} = \int_2^{10} \left[(12-x) - \left(1+\frac{9}{(x-1)}\right)\right] dx$$

$$= \int_2^{10} \left(11 - x - \frac{9}{(x-1)}\right) dx$$

$$= \left[11x - \frac{x^2}{2} - 9\ln|x-1|\right]_2^{10}$$

$$= \left(11(10) - \frac{(10)^2}{2} - 9\ln|10-1|\right) - \left(11(2) - \frac{(2)^2}{2} - 9\ln|2-1|\right)$$

$$= (110 - 50 - 9\ln|9|) - (22 - 2 - 9\ln|1|)$$

$$= 60 - 9\ln|9| - 20$$

$$= 40 - \ln|9|^9 \quad \text{As Required.}$$

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