AH 2002 PPQs

A1.

$$\left(\begin{array}{ccc|c}1&1&3&2\\2&1&1&2\\3&2&5&5\end{array}\right)\begin{array}{c}r_1\\r_2\\r_3\end{array} \to \begin{array}{c}6r_1\\3r_2\\2r_3\end{array}\left(\begin{array}{ccc|c}6&6&18&12\\6&3&3&6\\6&4&10&10\end{array}\right)\begin{array}{c}r_1\\r_2\\r_3\end{array}$$

$$\begin{array}{c}\frac{1}{6}r_1\\r_1-r_2\\r_1-r_3\end{array}\left(\begin{array}{ccc|c}1&1&3&2\\0&3&15&6\\0&2&8&2\end{array}\right)\begin{array}{c}r_1\\r_2\\r_3\end{array} \to \begin{array}{c}r_1\\\frac{1}{3}r_2\\\frac{1}{2}r_3\end{array}\left(\begin{array}{ccc|c}1&1&3&2\\0&1&5&2\\0&1&4&1\end{array}\right)\begin{array}{c}r_1\\r_2\\r_3\end{array}$$

$$\begin{array}{c}\\\\r_3-r_2\end{array}\left(\begin{array}{ccc|c}1&1&3&2\\0&1&5&2\\0&0&-1&-1\end{array}\right)\begin{array}{c}r_1\\r_2\\r_3\end{array} \Rightarrow$$

$-z = -1$

$\underline{\underline{z = 1}}$

$y + 5z = 2$

$y + 5 = 2$

$\therefore\ \underline{\underline{y = -3}}$

&

$x + y + 3z = 2$

$x - 3 + 3 = 2$

$\therefore\ \underline{\underline{x = 2}}$

⑤

$\therefore$ Solution is $(2, -3, 1)$

Q2. $z^4+4z^3+3z^2+4z+2=0$ Verify by showing 'fits' perfectly

$z=i$

$z^2=i^2=-1$

$z^3=i^3=i\times i^2=-i$

$z^4=i^2\times i^2=+1$

$$z^4+4z^3+3z^2+4z+2=0$$

$$(1)+4(-i)+3(-1)+4(i)+2$$

$$=1-4i-3+4i+2$$

$$=\underline{\underline{0}}$$

As no remainder ⇒ $z=i$ is a solution.

If $z=i$, then $\bar{z}=-i$ is conjugate pair.

factors are: $(z-i)=0$ & $(z+i)=0$

Multiplying: $(z-i)(z+i)=z^2-iz+iz-i^2$

$$=\underline{z^2+1}$$

⑤

Dividing into Quartic

$$\begin{array}{r|lllll}
 & z^2 & +4z & +2 & & \\ \hline
z^2+1 & z^4 & +4z^3 & +3z^2 & +4z & +2 \\
 & z^4 & & +z^2 & & \\ \hline
 & & 4z^3 & +2z^2 & +4z & +2 \\
 & & 4z^3 & & +4z & \\ \hline
 & & & 2z^2 & & +2 \\
 & & & 2z^2 & & +2 \\ \hline
\end{array}$$

$$z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-4\pm\sqrt{4^2-4\times1\times2}}{2\times1}$$

$$=\frac{-4\pm\sqrt{8}}{2}$$

$$=\frac{-4\pm2\sqrt{2}}{2}$$

$$\therefore z=\underline{\underline{-2\pm\sqrt{2}}}$$

$\therefore$ $z^4+4z^3+3z^2+4z+2=0$ has 4 solutions

$z=i$; $z=-i$; $z=-2+\sqrt{2}i$ & $z=-2-\sqrt{2}i$

Q3. $x = t^2 + t - 1$; $y = 2t^2 - t + 2$ $\Rightarrow$ $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$

$\dfrac{dx}{dt} = 2t + 1$ $\quad$ $\dfrac{dy}{dt} = 4t - 1$ ●

$= \dfrac{4t-1}{2t+1}$ ●

Pt $A(-1, 5) \Rightarrow x = -1$ & $y = 5$

If $x = t^2 + t - 1$

then $t^2 + t - 1 = -1$

$t^2 + t = 0$

$t(t+1) = 0$

$\swarrow \quad \downarrow$

$t = 0 ; t = -1$ ●

If $y = 2t^2 - t + 2$

$2t^2 - t + 2 = 5$

$2t^2 - t - 3 = 0$

$(2t - 3)(t + 1)$

$\downarrow \quad \downarrow$

$t = 3/2 ; t = -1$

As must satisfy both x & y

t must only have 1 solution

$t = -1$ ●

At $t = -1$ $y = 2 + 1 + 2 = 5$ ✓ true.

If $t = -1$ $\quad m = \dfrac{dy}{dx} = \dfrac{4t-1}{2t+1} = \dfrac{-4-1}{-2+1} = \dfrac{-5}{-1} = \underline{\underline{5}}$ ●

$A(-1, 5)$ & $m = 5$ $\quad (y - b) = m(x - a)$

$y - 5 = 5(x - (-1))$

$y - 5 = 5x + 5$

$\therefore y = 5x + 10$ ●

⑥

(or an alternative rearrangement)

AH 2002 PPQs

Q4. a) $f(x) = \sqrt{x}\,e^{-x}$

$$\boxed{\begin{array}{ll} u = x^{1/2} & v = e^{-x} \\ u' = \frac{1}{2}x^{-1/2} & v' = -e^{-x} \end{array}}$$

$$\boxed{f'(x) = u'v + uv'}$$

$$\therefore \quad f'(x) = \frac{1}{2\sqrt{x}} \cdot e^{-x} + \sqrt{x}.(-e^{-x})$$

$$= \frac{e^{-x}}{2\sqrt{x}} - \sqrt{x}\,e^{-x}$$

④

$$= \frac{e^{-x}}{2\sqrt{x}}(1-2x)$$

Q4b) $y = (x+1)^2(x+2)^{-4}$

$$\ln(y) = \ln\left((x+1)^2(x+2)^{-4}\right)$$

$$\ln y = \ln(x+1)^2 + \ln(x+2)^{-4}$$

$$\ln y = 2\ln(x+1) - 4\ln(x+2)$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{2}{x+1} - \frac{4}{x+2}$$

③

$$\underline{\frac{dy}{dx} = \left(\frac{2}{x+1} + \frac{-4}{x+2}\right) \times y}$$

Where $\underline{a = 2}$ & $\underline{b = -4}$

Q5. $\int_0^1 \ln(1+x)\,dx = \int_0^1 1.\ln(1+x)\,dx \leftarrow$ (Need to create 2 parts & can only Diff. $\ln(x)$)

$$\boxed{\int u'v = uv - \int uv'}$$

$$\boxed{\begin{array}{lll} \text{Let} & u=\ln(1+x) & v=x \\ & u'=\dfrac{1}{1+x} & v'=1 \end{array}}$$

$$\int_0^1 1.\ln(1+x)\,dx = \Big[x\ln(1+x)\Big]_0^1 - \int_0^1 \frac{x}{1+x}\,dx$$

$$= \Big[x\ln(1+x)\Big]_0^1 - \int_0^1 \left(\frac{x+1-1}{1+x}\right)dx$$ ⊛ or ↓

$$= \Big[x\ln(1+x)\Big]_0^1 - \int_0^1 \left(\frac{x+1}{1+x} - \frac{1}{1+x}\right)dx$$

$$= \Big[x\ln(1+x)\Big]_0^1 - \int_0^1 \left(1 - \frac{1}{1+x}\right)dx$$

$$= \Big[x\ln(1+x) - x + \ln(1+x)\Big]_0^1$$

$$= \Big(1\ln(2) - 1 + \ln(2)\Big) - \Big(0 - 0 + \ln(1)\Big)$$

$$= 2\ln 2 - 1$$ ⑤

Can split $\frac{x}{1+x}$ as above or

Divide $\Rightarrow$ $$\begin{array}{r|l} & \ \ 1 \\ \hline 1+x & x \\ & x+1 \\ \hline & \ -1 \end{array} \rightarrow \left(1 + \frac{-1}{1+x}\right)$$

Q6.

METHOD 1

$$x+2 = 2\tan\theta \quad \Rightarrow \quad x = 2\tan\theta - 2$$

$$x^2 = (2\tan\theta - 2)^2$$

$$= 4\tan^2\theta - 8\tan\theta + 4$$

If $x = 2\tan\theta - 2$

$$\frac{dx}{d\theta} = 2\sec^2\theta$$

$$\therefore \quad dx = 2\sec^2\theta\, d\theta$$

Remember to change EVERYTHING IN TERMS of x into θ

$$\int \frac{1}{x^2+4x+8}\,dx = \int \frac{1}{(2\tan\theta-2)^2 + 4(2\tan\theta-2)+8} \cdot 2\sec^2\theta\, d\theta$$

$$= \int \frac{2\sec^2\theta\, d\theta}{(4\tan^2\theta - 8\cancel{\tan\theta} + 4 + 8\cancel{\tan\theta} \cancel{-8+8})}$$

$$= 2\int \frac{\sec^2\theta\, d\theta}{4\tan^2\theta + 4}$$

⑤

$$= \frac{1}{2}\int \frac{\sec^2\theta}{(\tan^2\theta+1)}\, d\theta$$

$$= \frac{1}{2}\int \frac{\sec^2\theta\, d\theta}{\sec^2\theta}$$

$$= \frac{1}{2}\int 1\, d\theta$$

$$= \frac{1}{2}\theta + c$$

$$= \frac{1}{2}\left(\tan^{-1}\left(\frac{x+2}{2}\right)\right) + c$$

If $x+2 = 2\tan\theta$

$$\frac{x+2}{2} = \tan\theta$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{x+2}{2}\right)$$

Q6. **Method 2: If realise can complete square**

$$\int \frac{1}{x^2+4x+8}\,dx = \int \frac{dx}{(x+2)^2+4} = \int \frac{dx}{(x+2)^2+2^2}$$

(✱) See below for Version 3.

If $(x+2) = 2\tan\theta$

$x = 2\tan\theta - 2$

$\frac{dx}{d\theta} = 2\sec^2\theta$

$\therefore\ dx = 2\sec^2\theta\, d\theta$

$$= \int \frac{2\sec^2\theta\, d\theta}{(2\tan\theta)^2+2^2}$$

$$= \int \frac{2\sec^2\theta\, d\theta}{4\tan^2\theta+4}$$

$$= \frac{1}{2}\int \frac{\sec^2\theta}{(\tan^2\theta+1)}\,d\theta$$

$$= \frac{1}{2}\int \frac{\sec^2\theta}{\sec^2\theta}\,d\theta$$

$$= \frac{1}{2}\int d\theta$$

$$= \frac{1}{2}\theta + c$$

$$= \frac{1}{2}\left(\tan^{-1}\left(\frac{x+2}{2}\right)\right) + c$$

OR from $\int \frac{dx}{(x+2)^2+2^2} = \frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right) + c$ immediately.

Still get 3 (−2) marks if realise this (But not subst by θ as asked so 2 marks lost)

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

Q7. $\boxed{4^n - 1 \text{ is divisible by } 3 \ \forall n \in N}$

Let n=1 $4^1 - 1 = 3 = 3\times 1$ ✓ true for n=1

Assume true for n=k $\boxed{4^k - 1 = 3m}$ (*), where m is a positive integer and $4^k - 1$ is a multiple of 3.

Consider n=(k+1)

$$4^{k+1} - 1 = 4^k . 4^1 - 1 + 3 - 3$$

$$= 4^k . 4^1 - 4 + 3$$

$$= 4\left(\underbrace{4^k - 1}\right) + 3$$

(Use $4^k - 1 = 3m$ here) (*) $= 4(3m) + 3$

$$= 3(4m + 1)$$

⑤

$\Rightarrow$ $4^{k+1} - 1$ is a multiple of 3 & thus divisible by 3.

Statement:

As true for n=1, assumed true for n=k and by Proof of Mathematical Induction also true for n=k+1 $4^n - 1$ is divisible by 3, must be true $\forall n \in N$.

Q8.

$$\frac{x^2}{(x+1)^2} = \frac{x^2}{x^2+2x+1} \rightarrow \quad \begin{array}{r|l} & \quad 1 \\ \hline x^2+2x+1 & x^2 \\ & \underline{x^2+2x+1} \\ & \quad -2x-1 \end{array}$$

⊛ Remember to DIVIDE if Higher or SAME power on NUMERATOR

* Can work with ($x^2 = A(x+1)^2 + B(x+1) + C$)

$$\therefore \quad \frac{x^2}{(x+1)^2} = 1 + \frac{-2x-1}{(x+1)^2} = A + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$$ •

③

If $\frac{-2x-1}{(x+1)^2} = \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$

then $-2x-1 = B(x+1) + C$ •

Let $x=-1$: $2-1 = C \Rightarrow C=1$

Let $x=0$: $-1 = B+C$

$-1 = B+1 \Rightarrow B=-2$

So $\frac{x^2}{(x+1)^2} = 1 + \frac{-2}{(x+1)} + \frac{1}{(x+1)^2}$

Where $A=1$, $B=-2$, $C=1$ •

(i) Vertical Asymptote $(x+1) \neq 0 \Rightarrow x=-1$ •

Horizontal Asymptote is $y=1$ •

②

(ii) $f(x) = \frac{x^2}{(x+1)^2}$

$u = x^2 \qquad v = (x+1)^2$

$u' = 2x \qquad v' = 2(x+1)$

$$f'(x) = \frac{2x(x+1)^2 - 2x^2(x+1)}{(x+1)^4}$$

$$= \frac{2x(x+1)[(x+1)-x]}{(x+1)^4}$$

$$= \frac{2x}{(x+1)^3}$$ •

Using Quotient Rule

OR Use new layout for f(x)

$$f(x) = 1 - 2(x+1)^{-1} + (x+1)^{-2}$$

$$f'(x) = 2(x+1)^{-2} - 2(x+1)^{-3}$$

$$= \frac{2}{(x+1)^2} - \frac{2}{(x+1)^3}$$

$$= \frac{2(x+1)-2}{(x+1)^3}$$

$$= \frac{2x}{(x+1)^3}$$

Q8.

(ii)

$$f(x) = \frac{x^2}{(x+1)^2} = 1 - \frac{2}{(x+1)} + \frac{1}{(1+x)^2}$$

$$f'(x) = \frac{2x}{(x+1)^3} = 0$$

Stat Pt When $f'(x)=0$

is $x=0$

$$2x = 0$$

$$\therefore \quad \underline{\underline{x=0}}$$

y coord $\quad y = \frac{0^2}{(0+1)^2} = \frac{0}{1} = \underline{\underline{0}}$

$\therefore$ Stat Pt $\underline{(0,0)}$

Nature

$$\text{If } f'(x) = \frac{2x}{(x+1)^3}$$

$u=2x \quad v=(x+1)3$

$u'=2 \quad v'=3(x+1)^2$

$$f''(x) = \frac{2(x+1)^3 - 6x(x+1)^2}{((x+1)^3)^2}$$

$$= \frac{2(x+1)^2\left[(x+1)-3x\right]}{(x+1)^6}$$

$$= \frac{2(1-2x)}{(x+1)^4}$$

OR Use Nature Table

④

$$\underline{\text{At } x=0} \quad f''(0) = \frac{2(1-0)}{(0+1)^4} = \frac{2}{1} = 2 > 0 \Rightarrow \cup \text{ Min TPT } \underline{(0,0)}$$

Sketch

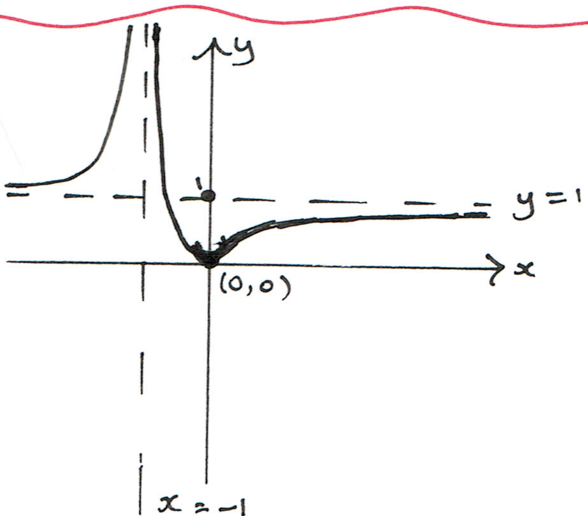
Asymptote Extremes

As $x \to -1^+ \quad y \to \infty^+$

$x \to -1^- \quad y \to \infty^+$

$x \to \infty^+ \quad y \to 1^-$

$x \to \infty^- \quad y \to 1^+$



Pts Annotated

Asymptotes & Sketch

②

Q9.-a)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-xy^2}{-x^2y} = \underline{\underline{\frac{y}{x}}}$$

If $\frac{dy}{dx} = \frac{y}{x}$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\ln|y| = \ln|x| + c$$

$$y = e^{\ln|x|+c}$$

$$= e^{\ln|x|} \cdot e^{c}$$

$$\therefore\ y = xe^{c}$$

⑤

$\underline{x=1;\ y=2}$ $\quad 2 = 1 \cdot e^{c} \Rightarrow \underline{e^{c} = 2}$

So $\quad y = xe^{c} \Rightarrow \underline{\underline{y = 2x}}$

$$\frac{dx}{dt} = -x^2y = -x^2(2x) = \underline{\underline{-2x^3}}$$

$$\frac{dx}{dt} = -2x^3$$

$$\int \frac{dx}{-2x^3} = \int dt$$

$$-\frac{1}{2}\int x^{-3}\,dx = \int dt$$

$$-\frac{1}{2}\left(\frac{x^{-2}}{-2}\right) = t + c$$

$$\frac{1}{4x^2} = t + c$$

$$\therefore\ \frac{1}{4(t+c)} = x^2$$

$$x = \pm\sqrt{\frac{1}{4(t+c)}}$$

(Need to find c)

$\underline{t=0;\ x=1}$

$$x = \pm\sqrt{\frac{1}{4(t+c)}}$$

$$1 = \pm\sqrt{\frac{1}{4c}}$$

$$1 = \frac{1}{4c}$$

$$4c = 1$$

$$c = 1/4$$

⑤

$$\therefore\ \underline{x = \pm\sqrt{\frac{1}{4t+1}}}$$

Q10.

$$\boxed{\begin{array}{l} S_n(x) = 1 + 2x + 3x^2 + \dots + nx^{n-1} \\ S_n(1) = 1 + 2 + 3 + \dots + n \Rightarrow \text{(Arithmetic Sum)} \end{array}}$$

$a=1\ ;\ d=1$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2 + (n-1)\right]$$

$$= \frac{n}{2}\left[n+1\right]$$

$$\boxed{\begin{array}{rl} S_n(x) & = 1 + 2x + 3x^2 + \dots + nx^{n-1} \\ -\quad xS_n(x) & = \quad x + 2x^2 + \dots + (n-1)x^{n-1} + nx^n \\ \hline \underbrace{S_n(x) - xS_n(x)} & = \left(1 + x + x^2 + \dots + x^{n-1}\right) - nx^n \\ \Rightarrow (1-x)S_n(x) & \end{array}}$$

$$\boxed{1 + x + x^2 + \dots \text{ is Geometric } a=1; r=x}$$

$$\boxed{S_n = \frac{a(1-r^n)}{1-r} = \frac{1.(1-x^n)}{1-x} = \frac{(1-x^n)}{(1-x)}}$$

⑥

$$(1-x)S_n(x) = \underbrace{\left(1 + x + x^2 + \dots + x^{n-1}\right)} - nx^n$$

$$(1-x)S_n(x) = \left(\frac{(1-x^n)}{(1-x)}\right) - nx^n$$

$$\therefore S_n(x) = \frac{(1-x^n)}{(1-x)^2} - \frac{nx^n}{(1-x)}$$

As $n \to \infty$

Q10

$$\lim_{n\to\infty}\left\{\frac{2}{3}+\frac{3}{3^2}+\frac{4}{3^3}+\dots+\frac{n}{3^{n-1}}+\frac{3}{2}\cdot\frac{n}{3^n}\right\}$$

Try to use findings to solve problem.

Recall $S_n(x) = 1+2x+3x^2+4x^3+\dots+nx^{n-1} = \frac{1-x^n}{(1-x)^2}-\frac{nx^n}{(1-x)}$

* If $(1+2x+3x^2+4x^3+\dots+nx^{n-1})$ can be rearranged to fit, can use findings

$$(\textcircled{1}+2x+3x^2+4x^3+\dots+nx^{n-1})-\textcircled{1}$$

$$= 2x+3x^2+4x^3+\dots+nx^{n-1}$$

If $x = 1/3$

$$\left\{2\left(\frac{1}{3}\right)+3\left(\frac{1}{3}\right)^2+4\left(\frac{1}{3}\right)^3+\dots+n\left(\frac{1}{3}\right)^{n-1}+\frac{3}{2}\cdot\frac{n}{3^n}\right\}$$

$$=\left[\left(1+2\left(\frac{1}{3}\right)+3\left(\frac{1}{3}\right)^2+\dots+n\left(\frac{1}{3}\right)^{n-1}\right)-1\right]+\frac{3}{2}\cdot\frac{n}{3^n}$$

$$=\left[\left(\frac{1-(1/3)^n}{(1-(1/3))^2}-\frac{n(1/3)^n}{(1-(1/3))}\right)-1\right]+\frac{3n}{2}\cdot\left(\frac{1}{3}\right)^n$$

③

$$=\left[\frac{1-\frac{1}{3^n}}{(2/3)^2}-\frac{n\left(\frac{1}{3^n}\right)}{(2/3)}-1\right]+\frac{3n}{2}\left(\frac{1}{3^n}\right)$$

$$=\frac{9\left(1-\frac{1}{3^n}\right)}{4}-\cancel{\frac{3n}{2}\cdot\left(\frac{1}{3^n}\right)}-1+\cancel{\frac{3n}{2}\cdot\left(\frac{1}{3^n}\right)}$$

$$=\frac{9}{4}\left(1-\frac{1}{3^n}\right)-1$$

As $n\to\infty$ $\lim\left\{\frac{9}{4}\left(1-\frac{1}{3^n}\right)-1\right\} = \frac{9}{4}\left(1-\frac{1}{\infty}\right)-1$

$$= 9/4\,(1-0)-1$$

$$= 5/4$$

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B2. $A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$ Prove $A^n = \begin{pmatrix} n+1 & n \\ -n & 1-n \end{pmatrix}$

Let n=1 $A^1 = \begin{pmatrix} 1+1 & 1 \\ -1 & 1-1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$ ✓ true for n=1

Assume true for n=k $A^K = \begin{pmatrix} K+1 & K \\ -K & 1-K \end{pmatrix}$, for some positive integer K.

Consider n=k+1 $A^{K+1} = A.A^K$

$$= \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} K+1 & K \\ -K & 1-K \end{pmatrix}$$

$$= \begin{pmatrix} 2(K+1)-K & 2K+1-K \\ -(K+1)+0 & -K+0 \end{pmatrix}$$

$$= \begin{pmatrix} 2K+2-K & (K+1) \\ -(K+1) & -K \end{pmatrix}$$

⑥

$$= \begin{pmatrix} K+2 & (K+1) \\ -(K+1) & -K+1-1 \end{pmatrix}$$

$$= \begin{pmatrix} (K+1)+1 & (K+1) \\ -(K+1) & 1-(K+1) \end{pmatrix}$$

Let $N=(K+1)$ $= \begin{pmatrix} N+1 & N \\ -N & 1-N \end{pmatrix}$

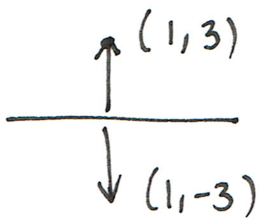
Statement

Thus as true for n=1, assumed true for n=k and by Proof of Mathematical Induction true for n=k+1, true $\forall n \in \mathbb{N}$ (or $\forall n \in \mathbb{Z}^+$)

AH 2002 PPQs

B4

Reflection in x-axis



$$A = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos 0 & \sin 0 \\ \sin 0 & -\cos 0 \end{pmatrix}$$

$$\Rightarrow \underline{A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}$$

Rotation of 30° A/C

$$B = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{pmatrix} = \underline{\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \sqrt{3}/2 \end{pmatrix}}$$

$$\therefore \quad (A)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$

$$(B)\begin{pmatrix} x \\ -y \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -1/2 \\ \frac{1}{2} & \sqrt{3}/2 \end{pmatrix}\begin{pmatrix} x \\ -y \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2}x + \frac{1}{2}y \\ \frac{1}{2}x - \frac{\sqrt{3}}{2}y \end{pmatrix}$$

$$\therefore \quad \left(\frac{\sqrt{3}x + y}{2}, \ \frac{x - \sqrt{3}y}{2} \right)$$

Where $k = \sqrt{3}$

④