Q1. $u_n = a + (n-1)d$

1st term, $a = 2$

20th term $u_{20} = a + (20-1)d$

$97 = 2 + 19d$

$95 = 19d$

$\therefore \quad d = 5$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $\quad a=2;\ d=5$

$S_{50} = \frac{50}{2}\left[2(2) + (50-1)\times 5\right]$

$= 25\left[4 + 49\times 5\right]$

$= 25\left[4 + 245\right]$

$= 25 \times 249$ $\quad$ 4

$= 6225$

Q2 a) $f(x) = \cos^{-1}(3x)$

$f'(x) = \frac{-1}{\sqrt{1-(3x)^2}} \cdot 3 = \frac{-3}{\sqrt{1-9x^2}}$ $\quad$ 2

Q2(b) $x = 2\sec\theta$

$= \dfrac{2}{\cos\theta}$

$= 2(\cos\theta)^{-1}$

$dx/d\theta = -2(\cos\theta)^{-2} \cdot -\sin\theta$

$= 2\sin\theta(\cos\theta)^{-2}$

$= \dfrac{2\sin\theta}{\cos^2\theta}$

$= 2\tan\theta\sec\theta$

$y = 3\sin\theta$

$dy/d\theta = 3\cos\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = 3\cos\theta \div \frac{2\sin\theta}{\cos^2\theta}$$

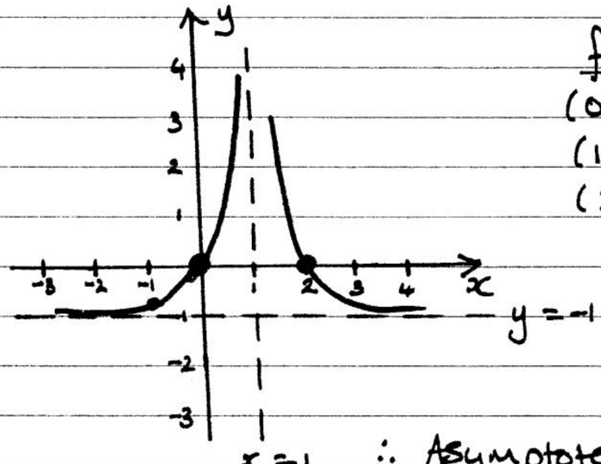
$$= 3\cos\theta \times \frac{\cos^2\theta}{2\sin\theta}$$

$$= \frac{3\cos^3\theta}{2\sin\theta}$$

$$= \frac{3}{2}\cot\theta\cos^2\theta$$

③

Q3.



$f(x)$	$-f(x)$	$-f(x+1)$
(0, 0·8)	(0, −0·8)	(−1, −0·8)
(1, 0)	(1, 0)	(0, 0)
(3, 0)	(3, 0)	(2, 0)

$-f(x) \rightarrow$ Reflect x-axis
$-f(x+1) \rightarrow$ left 1 place

$\therefore$ Asymptotes $y = -1$
$x = 1$

④

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Q4. $\dfrac{12x^2+20}{x(x^2+5)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+5}$

$A(x^2+5) + x(Bx+C) = 12x^2+20$

Let $x=0$: $A(0^2+5) + 0 = 12(0)^2+20$

$5A = 20$

$A = 4$

Let $x=1$: $A(1^2+5) + 1(B+C) = 12+20$

$6A + B + C = 32$

$24 + B + C = 32$

$B+C = 8$ — 1

Let $x=-1$: $A(1+5) - 1(-B+C) = 12+20$

$6A + B - C = 32$

$24 + B - C = 32$

$B - C = 8$ — 2

$B+C = 8$ — 1

$B-C = 8$ — 2

1+2: $2B = 16$

$B = 8$

$B+C=8$ — 1

$B-C=8$ — 2

1−2: $2C = 0$

$C = 0$

$\therefore \dfrac{12x^2+20}{x(x^2+5)} = \dfrac{4}{x} + \dfrac{8x}{x^2+5}$

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4

Q4. $\int_1^2 \frac{12x^2+20}{x(x^2+5)}\,dx = \int_1^2 \left(\frac{4}{x} + \frac{8x}{x^2+5}\right) dx$

$= \int_1^2 \frac{4}{x}\,dx + \int_1^2 \frac{8x}{x^2+5}\,dx$

$= \int_1^2 \frac{4}{x}\,dx + \int_6^9 \frac{8x}{u} \times \frac{du}{2x}$

$= \int_1^2 \frac{4}{x}\,dx + \int_6^9 \frac{4}{u}\,du$

Let $u = x^2+5$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

Let $u = x^2+5$

$\underline{x=1}$: $u = 1^2+5=6$

$\underline{x=2}$: $u = 2^2+5=9$

$= \left[4\ln|x|\right]_1^2 + \left[4\ln|u|\right]_6^9$

$= \left[4\ln|2| - 4\ln|1|\right] + \left[4\ln|9| - 4\ln|6|\right]$

$= 4\ln|2| + 4\ln\left|\frac{9}{6}\right|$

$= 4\left[\ln|2| + \ln\left|\frac{3}{2}\right|\right]$

$= \underline{\underline{4\ln|3|}}$

3

2008 AH

⑤

$$xy^2 + 3x^2y = 4$$

$$y^2 + 2xy\frac{dy}{dx} + 6xy + 3x^2\frac{dy}{dx} = 0$$

$$(y^2 + 6xy) + (2xy + 3x^2)\frac{dy}{dx} = 0$$

$$(2xy + 3x^2)\frac{dy}{dx} = -(y^2 + 6xy)$$

$$\therefore \quad \frac{dy}{dx} = \frac{-y(y+6x)}{x(2y+3x)}$$

③

When $x = 1$

$$xy^2 + 3x^2y = 4$$

$$y^2 + 3y = 4$$

$$y^2 + 3y - 4 = 0$$

$$(y+4)(y-1) = 0$$

$y = -4 \quad y = 1$, but $y > 0$

$\therefore y = 1$

$$\frac{dy}{dx} = \frac{-y(y+6x)}{x(2y+3x)} = \frac{-1(1+6)}{1(2+3)} = \frac{-7}{5}$$

③

$\boxed{m = -7/5 \text{ At } (1,1)}$

$$y - b = m(x - a)$$

$$y - 1 = \frac{-7}{5}(x - 1)$$

$$5y - 5 = -7x + 7$$

$$7x + 5y - 12 = 0$$

Q6 (a) $A = \begin{pmatrix} 1 & x \\ x & 4 \end{pmatrix}$

Singular if Det (A) = 0

$\therefore$ Det $A = (4 - x^2) = 0$

$4 = x^2$

$x = \pm 2$

2

Q6 (b) $x = 2$: $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

$A^2 = A.A. = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

$= \begin{pmatrix} 1+4 & 2+8 \\ 2+8 & 4+16 \end{pmatrix}$

$= \begin{pmatrix} 5 & 10 \\ 10 & 20 \end{pmatrix}$

$= 5\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

3

$\therefore \quad A^2 = 5A$ ie. $p = 5$

Thus, $A^4 = (A^2)^2 = (5A)^2 = 25A^2$

$= 25(5A)$

$= 125A$

$\therefore \quad q = 125$

Q7. $\boxed{\int uv' = uv - \int u'v}$

$u = 8x^2 \quad v = -\frac{\cos 4x}{4}$
$u' = 16x \quad v' = \sin 4x$

$\int 8x^2 \sin 4x \, dx$

$= \left(8x^2 \times -\frac{\cos 4x}{4}\right) - \int \left(16x \times -\frac{\cos 4x}{4}\right) dx$

$= -2x^2 \cos 4x - \int (-4x \cos 4x) dx$

$= -2x^2 \cos 4x + \int (4x \cos 4x) dx$

$u = 4x$
$u' = 4$
$v = \frac{\sin 4x}{4}$
$v' = \cos 4x$

$= -2x^2 \cos 4x + \left(4x \times \frac{\sin 4x}{4}\right) - \int \left(4 \times \frac{\sin 4x}{4}\right) dx$

$= -2x^2 \cos 4x + x \sin 4x - \int \sin 4x \, dx$

$= -2x^2 \cos 4x + x \sin 4x - \left(-\frac{\cos 4x}{4}\right) + C$

$= -2x^2 \cos 4x + x \sin 4x + \frac{1}{4} \cos 4x + C$

$= \left(\frac{1}{4} - 2x^2\right) \cos 4x + x \sin 4x + C$

5

Q8. $\boxed{\sum_{r=0}^{n} \binom{n}{r} (x)^{n-r} (y)^{r}}$ for $(x+y)^n$

$$\left(x^2+\frac{1}{x}\right)^{10} = \sum_{r=0}^{10} \binom{10}{r} (x^2)^{10-r} \cdot \left(\frac{1}{x}\right)^r$$

$$= \sum_{r=0}^{10} \binom{10}{r} (x^2)^{10-r} \cdot (x^{-1})^r$$

$$= \sum_{r=0}^{10} \binom{10}{r} x^{20-2r} \cdot x^{-r}$$

$$= \sum_{r=0}^{10} \binom{10}{r} x^{20-3r}$$

3

For term x^{14}:

$$x^{20-3r} = x^{14}$$
$$20-3r = 14$$
$$-3r = -6$$
$$r = 2$$

Thus Coeff when $r=2$: $\binom{10}{2} = \frac{10!}{2!\,8!}$

$$= \frac{10 \times 9 \times \cancel{8!}}{2 \times \cancel{8!}}$$

2

$$= 45$$

and term is $45x^{14}$

2008 AH

Q9. $f(x) = \tan x$

$f'(x) = \sec^2 x$ (1)

$$1 + \tan^2 x = 1 + \frac{\sin^2 x}{\cos^2 x}$$

$$= \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$ (1)

$$\boxed{\begin{array}{l} \text{If } 1 + \tan^2 x = \sec^2 x \\ \text{and } f(x) = \tan x \Rightarrow f'(x) = \sec^2 x \\ \text{then } f'(x) = 1 + \tan^2 x \text{ also} \end{array}}$$

$$\int \tan^2 x \, dx = \int \left((1 + \tan^2 x) - 1 \right) dx$$

$$= \int (\sec^2 x - 1)\, dx$$ (2)

$$= \tan x - x + c$$

Q10. velocity, $v = t^3 - 12t^2 + 32t$

Acceleration, $a = \frac{dv}{dt} = 3t^2 - 24t + 32$

At $t=0$: $\frac{dv}{dt} = 3(0)^2 - 24(0) + 32$

$\therefore a = 32$ at $t=0$ (1)

b) $s = \int v \, dt$

$= \int (t^3 - 12t^2 + 32t) \, dt$

$s = \frac{t^4}{4} - \frac{12t^3}{3} + \frac{32t^2}{2} + c$

$s = \frac{1}{4}t^4 - 4t^3 + 16t^2 + c$

At $t=0$, $s=0$

$0 = 0 - 0 + 0 + c \Rightarrow c = 0$

$\therefore s = \frac{1}{4}t^4 - 4t^3 + 16t^2$ (2)

$\frac{1}{4}t^4 - 4t^3 + 16t^2 = 0$

$t^4 - 16t^3 + 64t^2 = 0$

$t^2(t^2 - 16t + 64) = 0$ (2)

$t^2(t-8)^2 = 0$

$\downarrow \quad \downarrow$

$t=0 \quad t=8$

It returns to 0, $T=8$

2008 AH

Q11. If m^2 is divisible by 4, m is divisible by 4

By using a counter-example I will disprove this:

Let $m=2$ $\quad m^2=4$, which is divisible by 4

But if $m=2$, m is NOT divisible by 4

$\therefore$ A is FALSE

---

p is odd, Let $p = 2k+1$

&

q is even, Let $q = 2m$

Then $p^3 + q^2$

$= (2k+1) + (2m)^2$

$= (1)(2k)^3(1)^0 + (3)(2k)^2(1)^1 + (3)(2k)^1(1)^2 + (1)(2k)^0(1)^3 + (2m)^2$

$= 8k^3 + 12k^2 + 6k + 1 + 4m^2$

$= 2(4k^3 + 6k^2 + 3k + 2m^2) + 1$

$= 2N + 1 \Rightarrow$ Always odd

Let $N = (4k^3+6k^2+3k+2m^2)$ $\qquad$ 5

Thus B is TRUE

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Q12.

$u = x$
$u' = 1$
$v = \ln|2+x|$
$v' = \frac{1}{2+x}$

$$f(x) = x\ln|2+x|$$

$$f'(x) = \ln|2+x| + \frac{x}{2+x}$$

$u = x$
$u' = 1$
$v = 2+x$
$v' = 1$

$$f''(x) = \frac{1}{2+x} + \frac{2+x-x}{(2+x)^2}$$

$$= \frac{(2+x)+2}{(2+x)^2}$$

$$= \frac{4+x}{(2+x)^2}$$

$u = 4+x$
$u' = 1$
$v = (2+x)^2$
$v' = 2(2+x)$

$$f'''(x) = \frac{(2+x)^2 - 2(4+x)(2+x)}{(2+x)^4}$$

$$= \frac{4+4x+x^2-16-12x-2x^2}{(2+x)^4}$$

$$= \frac{-x^2-8x-12}{(2+x)^4}$$

$f(0) = \underline{\underline{0}}$

$f'(0) = \ln|2| + 0$
$= \underline{\underline{\ln|2|}}$

$f''(0) = \frac{4+0}{(2+0)^2}$
$= \frac{4}{4}$
$\therefore f''(0) = \underline{\underline{1}}$

$f'''(0) = \frac{-12}{(2)^4}$
$= \frac{-12}{16}$
$\therefore f'''(0) = \underline{\underline{\frac{-3}{4}}}$

3

$$\boxed{f(0) + f'(0)\frac{x}{1!} + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + \cdots}$$

$$\therefore x\ln|2+x| = (0) + (\ln|2|)\frac{x}{1!} + (1)\frac{x^2}{2!} - \left(\frac{3}{4}\right)\frac{x^3}{3!}$$

$$= \ln|2|x + \frac{x^2}{2} - \frac{x^3}{8}$$

2008 AH

Q12.

$$f(x) = x \ln|2-x|$$

$$f'(x) = \ln|2-x| + \frac{-x}{(2-x)}$$

$u = x \quad v = \ln|2-x|$
$u' = 1 \quad v' = \frac{1}{2-x} \cdot -1 = \frac{-1}{(2-x)}$

$$f''(x) = \frac{-1}{2-x} + \left(\frac{-2+x-x}{(2-x)^2}\right)$$

$u = -x \quad v = 2-x$
$u' = -1 \quad v' = -1$

$$= \frac{-(2-x)-2}{(2-x)^2}$$

$$= \frac{-4+x}{(2-x)^2}$$

$u = -4+x$
$u' = 1$
$v = (2-x)^2$
$v' = -2(2-x)$

$$f'''(x) = \frac{(2-x)^2 + 2(2-x)(x-4)}{(2-x)^4}$$

$$= \frac{(2-x) + 2(x-4)}{(2-x)^3}$$

$$= \frac{2-x+2x-8}{(2-x)^3}$$

$$= \frac{x-6}{(2-x)^3}$$

$f(0) = 0$
$f'(0) = \ln|2|$
$f''(0) = \frac{-4}{4} = -1$
$f'''(0) = \frac{-6}{8} = \frac{-3}{4}$

$$\therefore f(x) = x\ln(2-x)$$

$$= (0) + (\ln|2|)\frac{x}{1!} + (-1)\frac{x^2}{2!} + \left(\frac{-3}{4}\right)\frac{x^3}{3!}$$

$$\therefore x\ln|2-x| = \ln|2|x - \frac{x^2}{2} - \frac{x^3}{8}$$

2

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Q12.

$$\boxed{\begin{aligned} x\ln(2+x) &= \ln|2|x + \frac{x^2}{2} - \frac{x^3}{8} \\ x\ln(2-x) &= \ln|2|x - \frac{x^2}{2} - \frac{x^3}{8} \end{aligned}}$$

$$\begin{aligned} x\ln(4-x^2) &= x\left[\ln(2-x)(2+x)\right] \\ &= x\left[\ln(2-x) + \ln(2+x)\right] \\ &= x\ln(2-x) + x\ln(2+x) \\ &= \ln|2|x + \frac{x^2}{2} - \frac{x^3}{8} + \ln|2|x - \frac{x^2}{2} - \frac{x^3}{8} \\ &= 2\ln|2|x - \frac{2x^3}{8} \\ &= \ln|2|^2 x - \frac{x^3}{4} \\ &= \ln|4|x - \frac{x^3}{4} \end{aligned}$$

2

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Q13. $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2$

y_c $\quad m^2 - 3m + 2 = 0$

$(m-1)(m-2) = 0$

$m = 1 \qquad m = 2$

$\therefore \; y_c = Ae^{x} + Be^{2x}$

$y_p \quad y_p = ax^2 + bx + c$

$\frac{dy_p}{dx} = 2ax + b$

$\frac{d^2y_p}{dx^2} = 2a$

$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2$

$(2a) - 3(2ax+b) + 2(ax^2+bx+c) = 2x^2$

$2a - 6ax - 3b + 2ax^2 + 2bx + 2c = 2x^2$

$2ax^2 + (2b-6a)\,x + (2a-3b+2c) = 2x^2$

$\therefore \quad 2ax^2 = 2x^2 \qquad 2b - 6a = 0 \qquad 2a - 3b + 2c = 0$

$2a = 2 \qquad 2b - 6 = 0 \qquad 2 - 9 + 2c = 0$

$a = 1 \qquad 2b = 6 \qquad 2c = 7$

$b = 3 \qquad c = \frac{7}{2}$

$\therefore \quad y_p = x^2 + 3x + \frac{7}{2}$

7

$\therefore$ General Solution

$y = y_c + y_p = Ae^{x} + Be^{2x} + x^2 + 3x + 7/2$

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$$y = Ae^{x} + Be^{2x} + x^2 + 3x + \frac{7}{2}$$

$x=0$ $y=\frac{1}{2}$

$$\frac{1}{2} = Ae^{0} + Be^{0} + 0 + 0 + \frac{7}{2}$$

$$-\frac{6}{2} = A + B \Rightarrow A+B=-3 \quad — (1)$$

$$\frac{dy}{dx} = Ae^{x} + 2Be^{2x} + 2x + 3$$

$$1 = Ae^{0} + 2Be^{0} + 0 + 3$$

$x=0$

$dy/dx=1$ $\quad 1 = A + 2B + 3 \Rightarrow A+2B=-2 \quad — (2)$

$$A+B=-3 \quad — (1)$$
$$A+2B=-2 \quad — (2)$$

(1) − (2) $\quad -B=-1$

$$B=1$$

from (1) if $B=1$ $\quad A+B=-3$

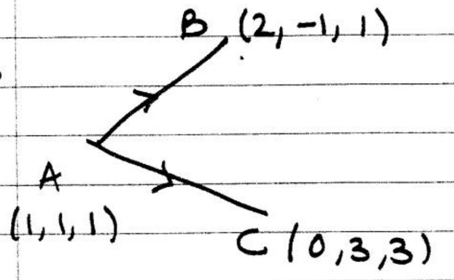
$$A+1=-3$$
$$A=-4$$

3

$$\therefore y = -4e^{x} + e^{2x} + x^2 + 3x + \frac{7}{2}$$

is particular solution

Q14.



$$\overrightarrow{AB} = \underline{b} - \underline{a} = \begin{pmatrix}2\\-1\\1\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\-2\\0\end{pmatrix}$$

$$\overrightarrow{AC} = \underline{c} - \underline{a} = \begin{pmatrix}0\\3\\3\end{pmatrix} - \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}-1\\2\\2\end{pmatrix}$$

$$\underline{n} = \overrightarrow{AB} \times \overrightarrow{AC}$$

$$= \begin{pmatrix}\underline{i} & \underline{j} & \underline{k}\\ 1 & -2 & 0\\ -1 & 2 & 2\end{pmatrix} = \underline{i}\begin{vmatrix}-2 & 0\\ 2 & 2\end{vmatrix} - \underline{j}\begin{vmatrix}1 & 0\\ -1 & 2\end{vmatrix} + \underline{k}\begin{vmatrix}1 & -2\\ -1 & 2\end{vmatrix}$$

$$= \underline{i}(-4-0) - \underline{j}(2-0) + \underline{k}(2-2)$$

$$= -4\underline{i} - 2\underline{j} + 0\underline{k}$$

$$\boxed{r \cdot n = a \cdot n}$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} \cdot \begin{pmatrix}-4\\-2\\0\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}-4\\-2\\0\end{pmatrix}$$

$$-4x - 2y + 0z = -4 - 2 + 0$$

3

$$-4x - 2y = -6$$

$$2x + y = 3$$

$$\underline{\pi_1:} \quad y = \underline{\underline{-2x+3}}$$

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Q14b) π_1 : $y = -2x + 3$

π_2 : $x + 3y - z = 2$

$(0, a, b) \Rightarrow x = 0$, $y = a$, $\cancel{z} = b$

from π_1 : $a = 0 + 3$

$\therefore \underline{\underline{a = 3}}$

from π_2 : $0 + 3a - b = 2$

$9 - b = 2$

$\underline{\underline{b = 7}}$

$\therefore$ π_1 & π_2 intersect at $\underline{\underline{(0, 3, 7)}}$

$y = -2x + 3$

$x + 3y - z = 2$

$\boxed{\text{Let } x = \lambda}$

Then $2\lambda + y = 3$ — ① Then $y = 3 - 2\lambda$.

$\lambda + 3y - z = 2$ — ②

① ×3 $6\lambda + 3y = 9$

② $\lambda + 3y - z = 2$

① − ② : $5\lambda + 0y + z = 7$

$\therefore$ $5\lambda + z = 7$

$\underline{\underline{z = 7 - 5\lambda}}$

$\therefore$ $x = \lambda$; $y = 3 - 2\lambda$; $z = 7 - 5\lambda$ (parametric

i.e. $\dfrac{x - 0}{1} = \dfrac{y - 3}{-2} = \dfrac{z - 7}{-5}$ $(= \lambda)$ (symmetric

4

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14(c) $\pi_1 : 2x + y = 3$ $\quad n_1 = (2, 1, 0)$

$\pi_2 : x + 3y - z = 2$ $\quad m = (1, 3, -1)$

$$\cos\theta = \frac{\underline{n} \cdot \underline{m}}{|\underline{n}||\underline{m}|}$$

$$= \frac{\begin{pmatrix}2\\1\\0\end{pmatrix} \cdot \begin{pmatrix}1\\3\\-1\end{pmatrix}}{\sqrt{4+1+0}\,\sqrt{1+9+1}}$$

$$= \frac{2+3+0}{\sqrt{5}\sqrt{11}}$$

$$= \frac{5}{\sqrt{55}}$$ 3

$\theta = \cos^{-1}(5/\sqrt{55})$

$\theta = \underline{47.6^\circ}$

Alternative for (b) $\quad \underset{\sim}{n} \times \underline{m} = \begin{pmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 1 & 0 \\ 1 & 3 & -1 \end{pmatrix}$

$$= \underline{i}\begin{vmatrix}1 & 0\\3 & -1\end{vmatrix} - \underline{j}\begin{vmatrix}2 & 0\\1 & -1\end{vmatrix} + \underline{k}\begin{vmatrix}2 & 1\\1 & 3\end{vmatrix}$$

Direction of line, D $= -\underline{i} + 2\underline{j} + 5\underline{k}$

$$r = a + \lambda d \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\3\\7\end{pmatrix} + \lambda\begin{pmatrix}-1\\2\\5\end{pmatrix}$$

$\therefore\ x = -\lambda;\ y = 3 + 2\lambda\ \&\ z = 7 + 5\lambda$

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Q15. $$f(x) = \frac{x}{\ln|x|}$$

$$\boxed{\begin{array}{ll} u = x & v = \ln|x| \\ u' = 1 & v' = 1/x \end{array}}$$

$$f'(x) = \frac{\ln|x| - x \times \frac{1}{x}}{[\ln|x|]^2}$$

$$= \frac{\ln|x| - 1}{[\ln|x|]^2}$$

$$f''(x) = \frac{\frac{1}{x}[\ln|x|]^2 - \frac{2\ln|x|}{x} \times (\ln|x| - 1)}{[\ln|x|]^4}$$

$$\boxed{\begin{array}{l} u = \ln|x| - 1 \\ u' = \frac{1}{x} \\ v = [\ln|x|]^2 \\ v' = 2[\ln|x|]' \cdot \frac{1}{x} \end{array}}$$

$$= \frac{\ln|x| - 2(\ln|x| - 1)}{x[\ln|x|]^3}$$

$$= \frac{\ln|x| - 2\ln|x| + 2}{x[\ln|x|]^3}$$

$$= \frac{2 - \ln|x|}{x[\ln|x|]^3}$$

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Q15

(b) $f'(x) = \dfrac{\ln|x| - 1}{[\ln|x|]^2}$ from (a).

For Stat Pts : set $f'(x) = 0$

$$\frac{\ln|x|-1}{[\ln|x|]^2} = 0$$

$$\ln|x| - 1 = 0$$

$$\ln|x| = 1$$

$$e^{\ln|x|} = e^1$$

$\therefore$ $x = e^1$ is Stat Pt x-value.

y coord :

If $f(x) = \dfrac{x}{\ln|x|} \Rightarrow y = \dfrac{e^1}{\ln|e|} = \dfrac{e}{1} = e$

$\therefore$ (e, e) is Stat Pt

Nature (Use $f''(x)$ as already found in (a))

$$f''(x) = \frac{2 - \ln|x|}{x\,[\ln|x|]^3}$$

$$\therefore\ f''(e) = \frac{2 - \ln|e|}{e\,[\ln|e|]^3} = \frac{2-1}{e[1]^3} = \frac{1}{e} > 0$$

If $f''(e) > 0$ $\cup$ $\therefore$ Min TPt (e, e)

Q15 (c) $\boxed{f''(x)=0 \quad \text{for pts of Inflexion}}$

$$f''(x) = \frac{2-\ln|x|}{x[\ln|x|]^3} = 0$$

$$2-\ln|x| = 0$$

$$2 = \ln|x|$$

$$e^2 = e^{\ln|x|}$$

$$\therefore \quad x = e^2$$

$$f(x) = \frac{x}{\ln|x|} \quad \text{at} \quad x = e^2$$

$$y = \frac{e^2}{\ln|e^2|} = \frac{e^2}{2\ln|e|} = \frac{e^2}{2}$$

$$\therefore \quad y = \tfrac{1}{2}e^2$$

and Coord for P.O.I is $(e^2, \frac{1}{2}e^2)$

Q16. $z^n = [\cos\theta + i\sin\theta]^n$
$= \cos(n\theta) + i\sin(n\theta)$

$\therefore \quad z^k = [\cos\theta + i\sin\theta]^k$
$= \cos(k\theta) + i\sin(k\theta)$

$\dfrac{1}{z^k} = z^{-k} = [\cos\theta + i\sin\theta]^{-k}$

$= \cos(-k\theta) + i\sin(-k\theta)$

$= \underline{\cos(k\theta) - i\sin(k\theta)}$ 3

OR

$\dfrac{1}{(\cos\theta + i\sin\theta)} \times \dfrac{(\cos\theta - i\sin\theta)}{(\cos\theta - i\sin\theta)}$

$= \dfrac{(\cos\theta - i\sin\theta)}{[\cos^2\theta - i\sin\theta\cos\theta + i\sin\theta\cos\theta - i^2\sin^2\theta]}$

$= \dfrac{\cos\theta - i\sin\theta}{(\cos^2 + \sin^2\theta) + 0}$

$= \dfrac{\cos\theta - i\sin\theta}{1}$

$= \underline{\underline{\cos\theta - i\sin\theta}}$

Q16 (cont..)

$$z^k = \cos(k\theta) + i\sin(k\theta)$$

$$z^{-k} = \cos(k\theta) - i\sin(k\theta)$$

$$z^k + \frac{1}{z^k} = \cos(k\theta) + i\sin(k\theta) + \cos(k\theta) - i\sin(k\theta)$$

$$= 2\cos(k\theta)$$

$$\therefore \quad \cos(k\theta) = \frac{1}{2}\left[z^k + \frac{1}{z^k}\right]$$

$$= \frac{1}{2}\left[z^k + z^{-k}\right]$$

$$z^k - \frac{1}{z^k} = \cos(k\theta) + i\sin(k\theta) - \left[\cos(k\theta) - i\sin(k\theta)\right]$$

$$= \cos(k\theta) + i\sin(k\theta) - \cos(k\theta) + i\sin(k\theta)$$

$$= 2i\sin(k\theta)$$

$$\therefore \quad \sin(k\theta) = \frac{1}{2i}\left(z^k - \frac{1}{z^k}\right)$$

$$\sin(k\theta) = \frac{1}{2i}\left(z^k - z^{-k}\right)$$

Q16. $\cos^2\theta \sin^2\theta = (\cos\theta \sin\theta)^2$

$$= \left[\frac{1}{2}(z^k + z^{-k}) \times \frac{1}{2i}(z^k - z^{-k})\right]^2$$

$$= \left[\frac{1}{4i}(z^k + z^{-k})(z^k - z^{-k})\right]^2$$

$$= \frac{1}{16i^2}(z^{2k} - z^{-2k})^2$$

$$= \frac{-1}{16}\left(z^{2k} - \frac{1}{z^{2k}}\right)^2$$

$$= \frac{-1}{16}\left(z^{4k} - 2\cancel{z^{2k}} \times \frac{1}{\cancel{z^{2k}}} + \frac{1}{z^{4k}}\right)$$

$$= \frac{-1}{16}\left(z^{4k} + \frac{1}{z^{4k}} - 2\right)$$

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$$= \frac{-1}{16}\left(2\cos 4\theta - 2\right)$$

$$= -\frac{1}{8}\cos 4\theta + \frac{1}{8}$$

$$= \frac{1}{8} - \frac{1}{8}\cos 4\theta$$

$\therefore \quad a = \frac{1}{8} \;\&\; b = \frac{-1}{8}$